EQI

INTEGRAL EQUATION PROGRAM FOR MODELING THE ACOUSTIC RADIATION AND SCATTERING OF IMMERSED THREE-DIMENSIONAL STRUCTURES

Version 6.0x

USER'S MANUAL

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ISEN - Institut Supérieur d'Électronique du Nord
Laboratoire d'Acoustique

The development of the integral equation code EQI was initiated by DCN-Toulon and DRET-Paris, and continues at the Laboratoire d'Acoustique of the Institut Supérieur d'Electronique du Nord (ISEN- Recherche, 90 rue Solférino, 59046 LILLE CEDEX, FRANCE).

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Chapter I: General Presentation

I.1 Introduction

EQI is an integral equation code that was specifically developed to simulate the acoustic behavior of three-dimensional structures immersed in a fluid of infinite dimensions (see Appendix I).

Three types of harmonic analyses are available:

- The radiation of a given displacement field
- The scattering of a given incident plane wave on a rigid target
- The fluid-structure interaction via the identification of impedance matrices used in a finite element code

Three computation modes are available:

- An axial symmetry mode: both the structure and the loading verify the symmetry about an axis of revolution
- A partial axial symmetry mode limited to scattering problems: the structure verify the symmetry about an axis of revolution and is subject to an oblique incident wave
- A fully three-dimensional mode with possible symmetries and anti-symmetries with respect to 1, 2 or 3 orthogonal planes

EQI computes the pressure on the external surface of the structure. From this result, the code can give the pressure in the fluid at a finite or infinite distance from the structure (monostatic and bistatic radiation and scattering patterns), the radiation impedance and the auto-scattering coefficients. **EQI** has been used for the modeling of various structures: flextensional transducers, acoustic antennas, submarines, and complex-shaped targets. The validation of the code was successfully carried out by numerous comparisons with analytical solutions, experimental measurements, and other numerical solutions. Most of these computations are described in the references shown in the next section.

Using adequate tools, two post-processing methods are available for a better understanding of the physical phenomena involved:

- The extraction of the free-body regime that allows for the identification of the resonances in the case of the fluid-structure coupling
- The generation of the polarization ellipses that allows the visualization of the particle trajectories in the fluid

EQI is the result of a research work carried out in large part by R. Bossut, A. Lavie, D. Morel, and C. Vanhille. Developments and tests of the code presently continue at ISEN.

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I.3 Organization of a Computation with EQI

A computation with **EQI** can be decomposed into several steps. This section describes these steps and refers, when necessary, to the corresponding chapters of this manual.

I.3.1 Model Definition

The data file used by **EQI** must be an accurate representation of the physical problem that is to be solved. Therefore, this file depends on the geometry of the structure, the surrounding fluid, the working frequency, the loading type, and the list of required results.

A precise description of the models offered by **EQI** is given in Chapter II.

At this stage of the model definition, it is possible to evaluate the size of the problem, which is mostly a function of the number of nodes in the mesh.

I.3.2 Mesh Generation

The mesh generation consists in describing precisely the geometry of the surface's structure using the elements available in **EQI**. Nodes are defined by their number and coordinates. These are then associated to form elements, by respecting the topological rules of the code.

It is important to find the best compromise between the mesh size and the accuracy of the results. Moreover, the mesh size may be limited according to the platform on which **EQI** is run. Finding the best compromise between mesh size and accuracy requires experience.

Information about the steps required for the mesh generation can be found in Paragraph III.5.

In most cases, the mesh generation can be partially or totally performed using mesh generation tools. Using such tools considerably simplifies the user's workload and limits his/her intervention to the definition of super-elements. Then, using automatic meshing functions, the mesh is obtained according to the required density and element types.

MOSAIQUE is the mesh generator typically used. Its use is described in the **ATILA** User's Manual. This manual also describes the graphic post-processing module **MDES**, which is used to display the mesh obtained.

I.3.3 Preparation of the Data File

The data file contains the definition of the nodes, the topology of the elements, the commands that are specific to the problem (fluid characteristics, frequency, resolution algorithm, theoretical formulation, analysis type, loading, etc.) and the list of the required results.

The commands are grouped in Paragraph III.4. An example of data file can be found in Chapter V.

A first verification of the data file can be done with the **EQILG** program (see Paragraph IV.1). The program has two functions: first, it tests the coherence of the data file, i.e. it verifies the orientation of the mesh elements and the compatibility of the commands; second, it computes the maximum frequency acceptable for the mesh, which depends on the size of the elements.

I.3.4 Running a Computation

After running a computation, the result file is available on the hard disk. Its content is described in Chapter V. In order to more easily perform graphical post-processing, some of the results can be directly stored in specific files (in particular, the near- and far-field pressures).

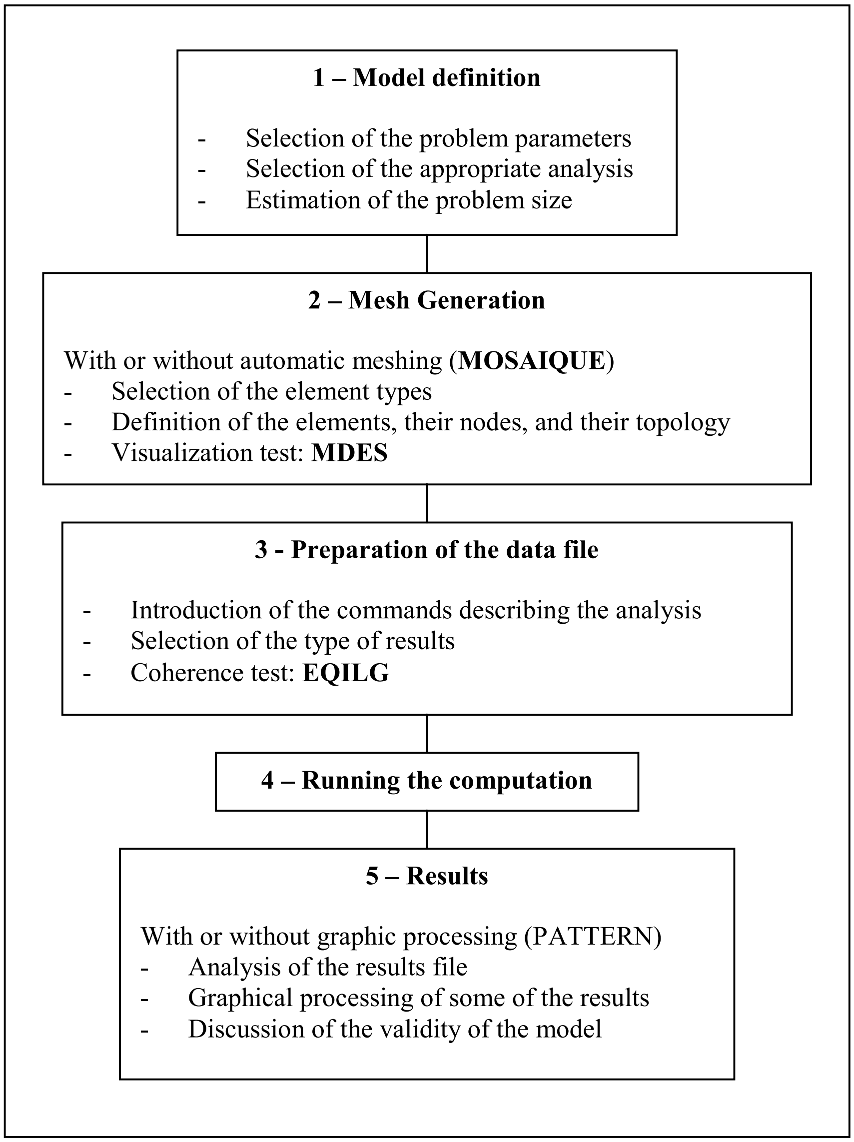
I.3.5 Results

The visualization of the results in the form of curves and diagrams is an important step for the physical understanding of the problem solved. Classical post-processing programs can be used. Also, the program **PATTERN** allows the creation of polar diagrams for the near- and far-field pressures. Its use is described in the **ATILA** User's Manual.

The validation of the model can be done by comparing the results obtained with **EQI** to results obtained through other methods (analytical or numerical), or to measurement results. Also, it is important to perform the point source test.

I.3.6 Summary

The five steps described above are schematically represented in Figure I.1.



1 – Model definition

- Selection of the problem parameters
- Selection of the appropriate analysis
- Estimation of the problem size

2 – Mesh Generation

With or without automatic meshing (**MOSAIQUE**)
- Selection of the element types
- Definition of the elements, their nodes, and their topology
- Visualization test: **MDES**

3 - Preparation of the data file

- Introduction of the commands describing the analysis
- Selection of the type of results
- Coherence test: **EQILG**

4 – Running the computation

5 – Results

With or without graphic processing (PATTERN)
- Analysis of the results file
- Graphical processing of some of the results
- Discussion of the validity of the model

Figure I.1: Organization of a computation with EQI.

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Chapter II: Description of the Fields of Application

II.1 General Formulation

This section describes the general formulation used in the **EQI** code to model problems of acoustic diffusion. The integral formulation used in **EQI** consists of the external Helmholtz representation also called the integral equation of Helmholtz-Kirchhoff. For an irregular frequency (eigenvalue of the associated Dirichlet interior problem), the system of integral equations theoretically allows an infinity of solutions. This system must then be over-determined by supplementary equations in order to guarantee the uniqueness of the solution. The discretization of these equations results in a linear system of over-determined or quadratic equations according to whether the null-field equations are incorporated into the system or not.

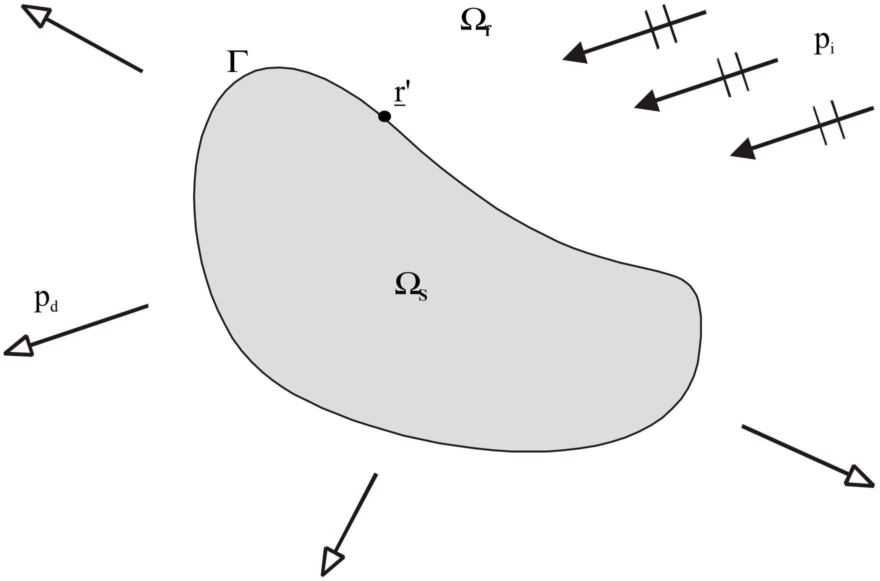


Figure II.1: Geometry of the problem.

Γ represents the structure's external surface, Ω_f the fluid domain with infinite dimensions, $\underline{r}$' a point along Γ, p_i the incident plane pressure, and p_d the diffused pressure. This pressure p_d is the sum of the rigid scattered pressure p_{dr}, i.e. the pressure generated by the scattering of the incident wave on the structure whose surface is assumed to be perfectly rigid, and the pressure only due to the vibration of the structure's surface. The total pressure p is given by $p = p_i + p_d$. In the case of a radiation problem, we have $p = p_r$.

II.1.1 Integral Formulation

By imposing a time dependence $e^{-j\omega t}$ of the variables and the positive orientation of the normal to Γ in the outward direction, the base integral equation is:

$$p_i(\underline{r}) + \iint_{\Gamma} \left(p(\underline{r}') \frac{\partial g(\underline{r},\underline{r}')}{\partial n'} - \frac{\partial p(\underline{r}')}{\partial n'} g(\underline{r},\underline{r}') \right) d\underline{r}' = \begin{cases} \dfrac{\alpha(\underline{r})}{4\pi} p(\underline{r}) & \underline{r} \in \Gamma \\ p(\underline{r}) & \underline{r} \in \Omega_f \end{cases} \tag{1}$$

where $\alpha(\underline{r})$ is the solid angle at point $\underline{r}$ seen from outside $\partial/\partial n'$ is the normal derivative at the point of integration $\underline{r}'$, and g Green's function associated to the three-dimensional infinite space such that:

$$g(\underline{r},\underline{r}') = \frac{1}{4\pi} \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} \tag{2}$$

with $|\underline{r}-\underline{r}'|$ as the distance between the computation points $\underline{r}$ and $\underline{r}'$.

The discretization of the integral equation on Γ consists in subdividing the structure into elements interconnected by nodes. Then, for each element, the pressure and pressure normal derivative fields are associated to the nodal values by interpolation functions.

In **EQI**, the use of quadratic isoparametric elements allows for a parabolic description of the element geometry and of the pressure and normal pressure derivative fields on each element. This description guarantees a good approximation of curved surfaces and results in the derivation of a linear system containing N equations and N unknowns, where N is the number of mesh nodes. The matrix form of the system is:

$$[A]\{p\} = [B]\left\{\frac{\partial p}{\partial n}\right\} - \{p_i\} \tag{3}$$

The [B] and [A] matrices come from the integration of Green's function and its normal derivative, averaged by the interpolation functions. The {p} and {∂p/∂n} vectors contain the nodal pressures and their derivatives that solve the system.

The vector u_n of the normal displacements at the surface can be introduced by the relationship:

$$\frac{\partial p(\underline{r})}{\partial n} = \rho\omega^2 u_n(\underline{r}) \tag{4}$$

where ρ is the density of the fluid and ω the frequency. The solution of the linear system (3) gives the pressure at the surface nodes from which the integral equation in Γ_f allows, for finite distances, the determination of the pressure in all points of the fluid.

In spherical coordinates, the directivity factor $f_0(\theta, \varphi)$ that characterizes the far-field pressure is such that:

$$p(\underline{r}) - p_i(\underline{r}) \sim f_0(\theta, \varphi)\frac{e^{ikr}}{r} \tag{5}$$

where r is the radius of the computation sphere and k the wave number. Taking the limit of the integral equation in Γ_f allows the identification of the directivity factor:

$$f_0(\theta, \varphi) = \frac{ik}{4\pi}\iint_{\Gamma} p(\underline{r}')\frac{\underline{r}\cdot\underline{n}'}{r} e^{-ik\frac{\underline{r}-\underline{r}'}{r}} d\underline{r}' - \frac{1}{4\pi}\iint_{\Gamma} \frac{\partial p(\underline{r}')}{\partial n'} e^{-ik\frac{\underline{r}-\underline{r}'}{r}} d\underline{r}' \tag{6}$$

By analogy to the matrix expression of system (3), we have:

$$f_0(\theta, \varphi) = [A_\infty]\{p\} - [B_\infty]\left\{\frac{\partial p}{\partial n}\right\} \tag{7}$$

II.1.2 Null-Field Equations

Assuming that the operating frequency coincides with an irregular frequency, system (3) theoretically allows an infinity of solutions. Several techniques of over-determination guarantee the uniqueness of the solution. Among them, Jones' method /D.S. JONES "Integral Equations for the Exterior Problem", Q. J. Mech. Appl. Math., XXVII, 129-142 (1973)/ presents the advantage of offering a criterion that indicates the sufficient number of supplementary null-field equations. The null-field equations are given by:

$$\iint_{\Gamma} p(\underline{r}) \frac{\partial \Psi_m^l(\underline{r})}{\partial n} d\underline{r} - \iint_{\Gamma} \frac{\partial p(\underline{r})}{\partial n} \Psi_m^l(\underline{r}) d\underline{r} = -4\pi c_m^l \quad m = 0,\ldots,M \quad |l| \le m \tag{6}$$

where Ψ_m^l is the spherical wave function and c_m^l is one of the coefficients of the decomposition of the incident pressure in a series of spherical waves converging and regular at the origin.

The discretization of this equation in the **EQI** code can be done in two ways:

- By using isoparametric elements everywhere
- By using isoparametric elements everywhere except for the pressure, which is approximated by a double Fourier series

II.1.2.1 Using Isoparametric Elements Everywhere

The discretization can be written:

$$[L]\{p\} = [F]\left\{\frac{\partial p}{\partial n}\right\} - \{c\} \tag{9}$$

The [F] and [L] matrices come from the integration of the spherical wave and its normal derivative averaged by the interpolation functions. The {c} vector contains the coefficients of the incident wave decomposition.

II.1.2.2 Using Isoparametric Elements Everywhere Except for the Pressure, which is Approximated by a Double Fourier Series

The double Fourier series is such that:

$$p(\underline{r}) = \sum_{p=0}^{P} \sum_{q=0}^{Q} a_p^q \cos(p\theta) e^{-iq\varphi} \tag{10}$$

It becomes:

$$[L']\{a\} = [F]\left\{\frac{\partial p}{\partial n}\right\} - \{c\} \tag{11}$$

The [L′] matrix comes from the integration of the product of the spherical wave function with the trigonometric functions that appear in the series representing the pressure. The {a} vector contains the coefficients of the decomposition (10).

Note: Comparing to the isoparametric approach in equation (9), the decomposition (10) makes it possible to select the level of precision, but should be reserved for experienced users. On the other hand, this decomposition increases the difficulty of the numerical treatment for the matrix in the first member.

Note: Each one of equations (9) and (10) can be used to compute directly or indirectly the pressure on Γ. In this case, they are numerically inadequate for eccentric structures. However, the use of equations (9) and (11) at several origins scattered in the structure allows a better description of the surface by spheres centered at these origins and inscribed within the geometry. This results in higher accuracy for elongated bodies. Classically, these are more efficiently used to over-determine the system of integral equations when the working frequency is an irregular frequency. In this case and assuming an approximation of the pressure by equation (10), a representation change matrix named [S] obtained from the inversion of the double series is necessary:

$$\{a\} = [S]\{p\} \tag{12}$$

II.2 Radiation of a Vibrating Structure

In this analysis, the displacement field on the surface is given and no incident wave excites the structure. The total pressure is then $p = p_r$. The system of integral equations (3) is reduced to:

$$[A]\{p\} = [B]\left\{\frac{\partial p}{\partial n}\right\} \tag{13}$$

The user provides the displacement values at the mesh nodes and the code computes the pressure at the surface. The solution of this system is based on Gauss' elimination method. It can be performed dynamically in memory or by swapping on the hard disk, according to the platform used.

From the surface pressure, the possible results are the computation of the radiation impedance, the pressure at any point of the fluid (at a finite distance), and the directivity diagrams.

In case of irregular frequencies, the previous system can be over-determined by the following null-field equations:

$$[L]\{p\} = [F]\left\{\frac{\partial p}{\partial n}\right\} \tag{14}$$

or:

$$[L'][S]\{p\} = [F]\left\{\frac{\partial p}{\partial n}\right\} \tag{15}$$

deduced from equations (9) and (11) according to the pressure approximation selected. The solution is found by a method based on the least squares method.

<u>Note:</u> It is possible, in the same run, to simultaneously treat other radiation problems by the introduction of different displacement fields, and other scattering problems by the introduction of incident plane waves. In this last case, the solutions from radiation and scattering problems allow for the computation of the auto-scattering coefficients (see Appendix II).

II.3 Scattering by a Rigid Structure

In this analysis, one or more incident waves converge to the structure whose surface is assumed to be perfectly rigid. The normal derivative of the pressure is zero by definition. The total pressure is therefore $p = p_i + p_{dr}$. The system of integral equations (3) is reduced to:

$$[A]\{p\} = -\{p_i\} \tag{16}$$

The user defines one or more incident waves and the code computes the surface pressure. This system solution is based on Gauss' elimination method. It can be carried out dynamically in memory or by swapping on the hard disk.

From the surface pressure, the possible results are the computation of the pressure at any point of the fluid (at a finite distance) and the monostatic and bistatic directivity diagrams.

In case of irregular frequencies, the previous system can be over-determined by the following null-field equations:

$$[L]\{p\} = -\{c\} \tag{14}$$

or:

$$[L'][S]\{p\} = -\{c\} \tag{15}$$

deduced from equations (9) and (11) according to the pressure approximation selected. The solution is found by a method based on the least squares method.

Note: It is possible, in the same run, to simultaneously treat scattering problems by the introduction of incident plane waves, and other radiation problems by the introduction of different displacement fields. In this last case, the solutions from radiation and scattering problems allow for the computation of the auto-scattering coefficients (see Appendix II).

II.4 Fluid-Structure Interaction

In this analysis, the user defines one or more incident waves converging on the structure. The total pressure is therefore $p = p_i + p_{dr} + p_r$. In the case of a radiation problem, no loading can be defined and $p = p_r$. In order to highlight the impedance matrices, the system of integral equations (3) is transformed into:

$$\{p\} = \rho\omega^2 [A]^{-1} [B] \{U_n\} - [A]^{-1} \{p_i\} \tag{19}$$

The term $[A]^{-1}\{p_i\}$ does not exist in the case of a radiation problem. By introducing the three components $\{U_x\}$, $\{U_y\}$, and $\{U_z\}$ of the displacement, and $\{n_x\}$, $\{n_y\}$, and $\{n_z\}$ of the normal at the nodes, we obtain:

$$\{U_n\} = \{U_x\}\{n_x\} + \{U_y\}\{n_y\} + \{U_z\}\{n_z\} \tag{20}$$

After projecting the [B] matrix on the three directions $\{n_x\}$, $\{n_y\}$, and $\{n_z\}$, we obtain three matrices $[B_x]$, $[B_y]$, and $[B_z]$ such that:

$$\{p\} = \rho\omega^2 \left([A]^{-1} [B_x] \{U_x\} + [A]^{-1} [B_y] \{U_y\} + [A]^{-1} [B_z] \{U_z\} \right) - [A]^{-1} \{p_i\} \tag{21}$$

The vector of known nodal pressures is therefore expressed as a function of the unknown nodal displacement vector.

The use of this expression in a program based on the finite element method applied to the structural problem is done via the impedance matrices, i.e. $[A]^{-1}[B_x]$, $[A]^{-1}[B_y]$, and $[A]^{-1}[B_z]$, and, when necessary, $[A]^{-1}\{p_i\}$.

The **EQIATI** (see Paragraph IV.2) code performs the necessary operations to obtain the impedance matrices. The finite element code allows the computation of the displacements from which, after substitution in the previous expression, the pressure on Γ is computed. At the present time, the coupling between **EQI** and **ATILA** is operational, and handled automatically on PC platforms.

In case of irregular frequencies, the previous system of integral equations, before pre-multiplication by $[A]^{-1}$, can be over-determined by the following null-field equations:

$$[L]\{p\} = \rho\omega^2\left([F_x]\{U_x\} + [F_y]\{U_y\} + [F_z]\{U_z\}\right) - \{c\} \tag{22}$$

or:

$$[L'][S]\{p\} = \rho\omega^2\left([F_x]\{U_x\} + [F_y]\{U_y\} + [F_z]\{U_z\}\right) - \{c\} \tag{23}$$

according to the pressure approximation method where the $[F_x]$, $[F_y]$, and $[F_z]$ matrices are the projection of [F] on the three directions $\{n_x\}$, $\{n_y\}$, and $\{n_z\}$. The term {c} does not exist in the case of a radiation problem. The resulting over-determined system is multiplied by the transpose of the matrix in the first term to obtain square matrices. Then, the previous process can resume.

II.5 Scattering by an Axially Symmetric Structure under Oblique Incidence

In the case of the last two applications (scattering by a rigid structure, fluid-structure interaction) and under the condition that the structure presents an axial symmetry and that it is subject to an oblique incident wave, a special treatment based on a Fourier series decomposition of the fields (incident and total pressures, and displacement) about the axis of revolution is available. Its principle relies on the following developments:

- Decomposition of the incident pressure in the cylindrical coordinate system
- Decomposition of the pressure and normal displacement in Fourier series

II.5.1.1 Decomposition of the incident pressure in the cylindrical coordinate system

In this case:

$$p(\underline{r}) = p_0 e^{ikz\cos\theta_0} \sum_{q=0}^{\infty} \varepsilon_q i^q J_q(k\rho\sin\theta_0)\cos(q\varphi) \tag{24}$$

where θ_0 defines the direction of the incident plane wave, p_0 its amplitude, and J_q Bessel's function of the first kind and degree q.

II.5.1.2 Decomposition of the pressure and normal displacement in Fourier series

The series are:

$$p(\underline{r}) = \sum_{q=0}^{\infty} p_q(\rho, z)\cos(q\varphi) \tag{25.a}$$

$$u_n(\underline{r}) = \sum_{q=0}^{\infty} u_{nq}(\rho, z)\cos(q\varphi) \tag{25.b}$$

The relationships (24) and (25) are then substituted into equations (1) and (8), and using:

$$c_m^l = \frac{-i}{kj_m(kR')}\int_0^{2\pi}\left(\int_0^{\pi} p_i(\underline{r}')Y_m^l(\theta',\varphi')\sin\theta' d\theta'\right)d\varphi' \qquad (26)$$

where $\underline{r}'$ is a point on a sphere of radius R′ tending to 0, j_m is the spherical Bessel function of the first kind and order m, and Y_m^l the spherical harmonic of order m and degree l. After obtaining the developments, the **EQI** code computes the coefficients p_q and u_{nq} for the harmonic q. p and u_n are directly obtained from the series (25) via the **OBLIC** code (see Paragraph IV.4).

Note: The decomposition into cylindrical harmonics is totally transparent to the user. It is only required to start a classical computation and simply indicate the number of the harmonic considered.

II.6 Radiation of a Piston in a Rigid Baffle

In this analysis, a plane piston of any shape is inserted into a rigid baffle of infinite planar dimension. The displacement field on the active surface of the piston is given. The integral formulation reduces then to a simple summation:

$$p(\underline{r}) = \frac{-i}{2\pi} \iint_{\Gamma} \frac{\partial}{\partial n'} p(\underline{r}') \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} d\underline{r}' \quad \underline{r} \in \Omega_f \cup \Gamma \tag{27}$$

where Γ_f is the semi-infinite space limited by the baffle’s plane. The user indicates the displacement values at the mesh nodes restricted to the active surface of the piston. The code directly computes the pressure at the surface, in the fluid (at finite distances), and the directivity diagrams. The computation of the radiation impedance is possible from the surface pressure (see Appendix II).

Note: It is possible, in the same run, to simultaneously treat other radiation problems by introducing different displacement fields. The formulation (27) is efficient: it does not include irregular frequencies, and no linear system solution or matrix inversion is performed.

EQI

ISEN-LILLE

Chapter III: Preparation of the Data File

III.1 Conventions

III.1.1 Units

Table III.1 shows the physical quantities and units considered.

Quantity	Units
Distance	meter (m)
Frequency	Hertz (Hz)
Pressure	Pascal (Pa)
Angle	degree (º)

Table III.1: Physical quantities and units

III.1.2 Coordinates

The definition of coordinates follows the classical definitions, and is recalled in Fig. III.1:

- Cartesian coordinates x, y & z
- Cylindrical coordinates ρ, φ & z
- Spherical coordinates r, θ et φ

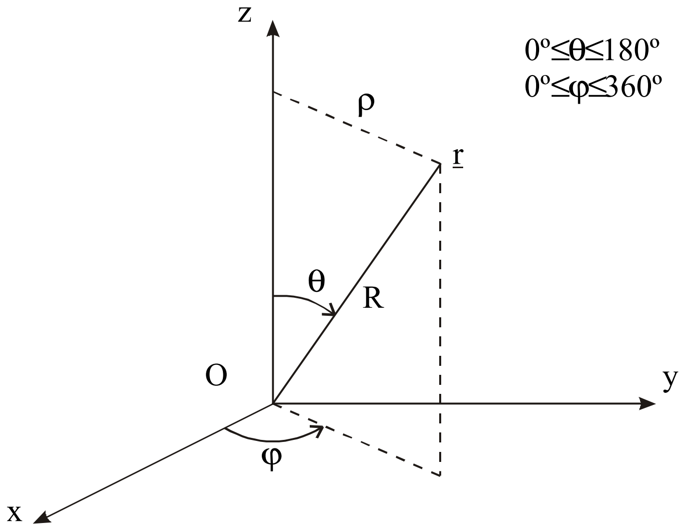


Figure III.1: System coordinates

III.1.3 Orientation of Incident Waves

The orientation of the incident plane wave is given by spherical angles θ_0 and φ_0, as shown in Fig. III.2.

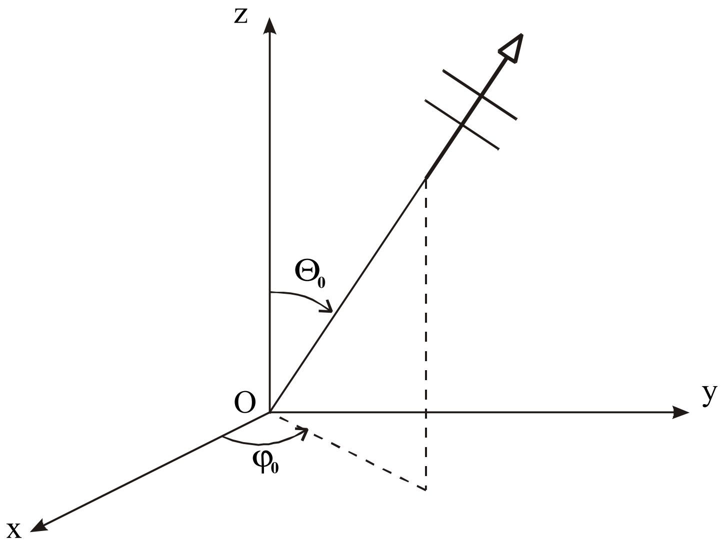


Figure III.2: Definition of the orientation of an incident plane wave

Example: $\theta_0 = 90^\circ$ and $\varphi_0 = 180^\circ$ define a wave propagating in the negative x direction.

III.1.4 Axis of Revolution

In the case of structures displaying an axial symmetry, the axis of revolution must be the z axis, and the reference plane must be the xOz plane. This is illustrated in Fig. III.3.

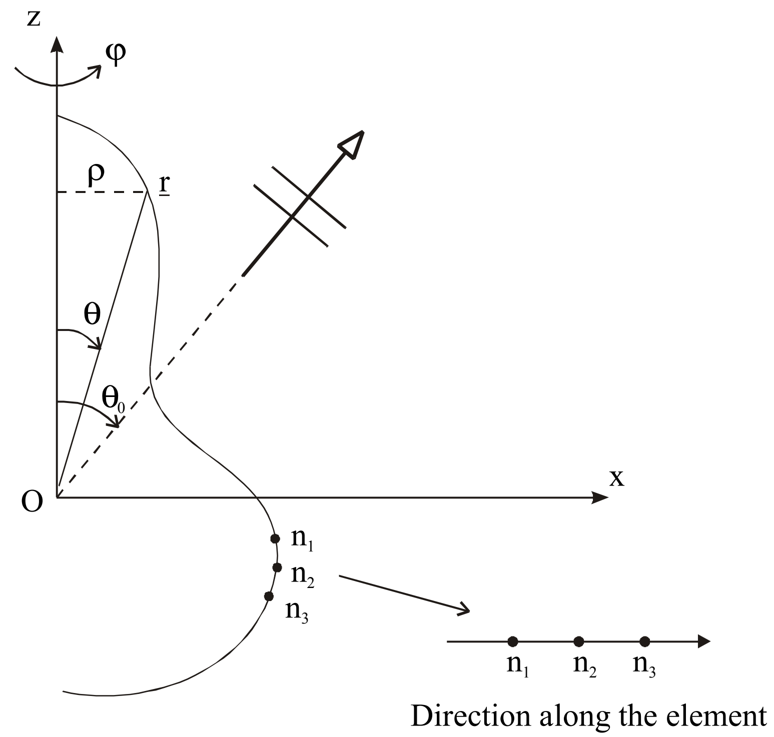


Figure III.3: Geometry of a structure with an axial symmetry

III.1.5 Structure of a Data File

The content of a data file for **EQI** is composed of five sections:

1. Header
2. Mesh description
3. List of commands
4. Loads definition
5. Postprocessing options

III.2 Description of Examples Cases

Two example cases are proposed: the first concerns a cylinder with quasi-elliptical cross sections (example case 1), and the second deals with a cylinder with hemispherical end caps (example case 2). These examples are used to illustrate the commands of **EQI** (section III.4), and a full listing of the files, including results, is given in Chapter V.

III.2.1 Example Case 1

The structure considered is shown in Fig. III.4. The dimensions are given in Fig. III.5.

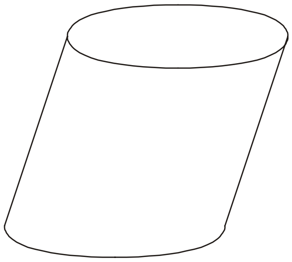


Figure III.4: Structure for example case 1

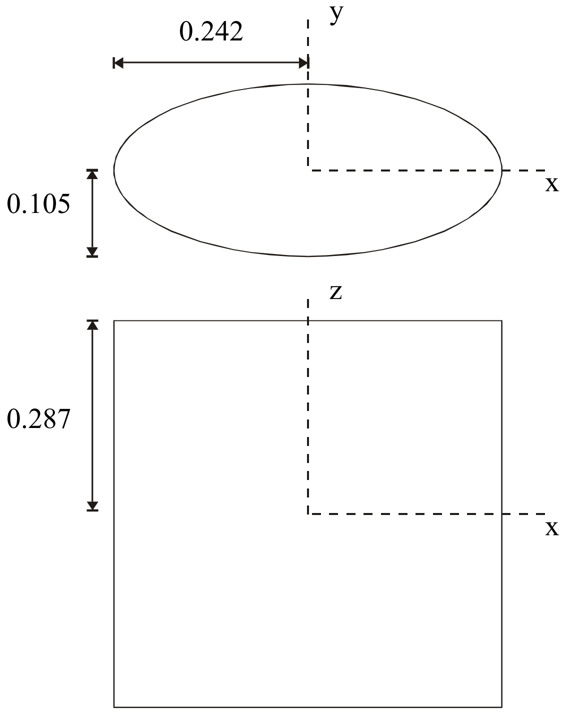


Figure III.5: Dimensions of the structure for example case 1

Three types of loads are mixed in this case:

- An incident plane wave of amplitude 1 µPa oriented in the positive x direction
- The incident plane waves of amplitude 1 µPa that correspond to the far field directivity diagram in the xOz plane, in various directions between the x and z axes
- A point source of amplitude 1 µPa located at the point of coordinates x = 0.05, y = 0.0 and z = 0.1.

The model must be tridimensional. It must contain the three planes of symmetry of the structure (xOy, xOz and yOz) and the plane of symmetry of the loads (xOz). **EQI** is then capable of using a decomposition into elementary problems according to Table III.2.

	Elementary problem 1	**Elementary problem 2**	**Elementary problem 3**	**Elementary problem 4**
xOz	symmetry	Symmetry	symmetry	symmetry
xOy	symmetry	Symmetry	antisymmetry	antisymmetry
yOz	symmetry	Antisymmetry	symmetry	antisymmetry

Table III.2: Elementary problems composing the example case 1

For each elementary problem, the mesh is reduced to a quarter of the initial mesh, i.e. one eighth of the surface. The final solution if the sum of the solution for each elementary problem.

The mesh used is shown in Fig. III.6. It is limited to one half of the total surface. During the decomposition in elementary problems, **EQI** automatically divides this mesh into four parts, i.e. in eighth of the surface.

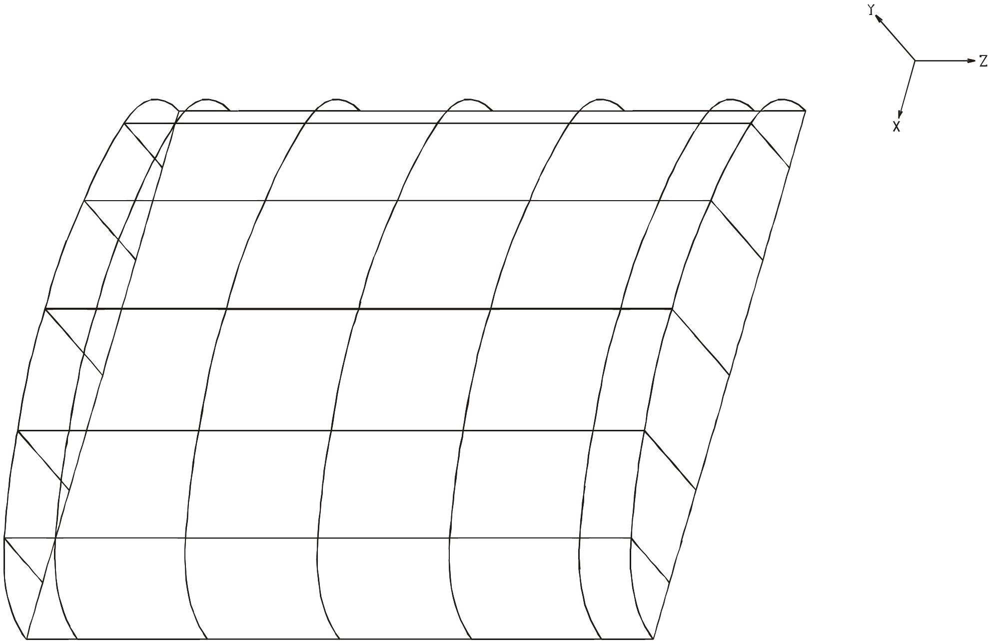


Figure III.6: Mesh used in example case 1

III.2.2 Example Case 2

The structure for this example case is shown in Fig. III.7. The dimensions are given in Fig. III.8.

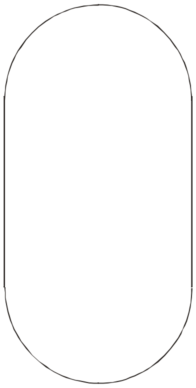


Figure III.7: Structure for the example case 2

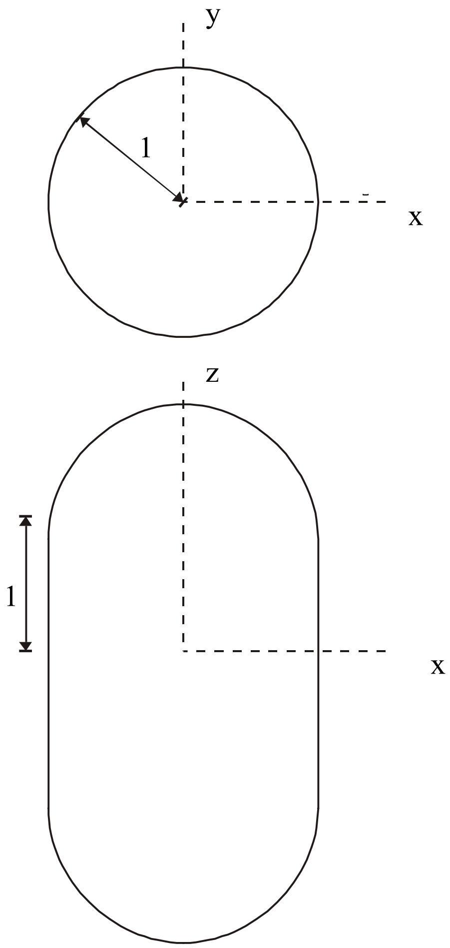


Figure III.8: Dimensions of the structure for example case 2

Three types of loads are mixed in this case:

- An incident plane wave of amplitude 1 µPa oriented in the negative z direction
- An in-phase displacement field of 1 nm applied to the vertical wall of the structure
- A point source of amplitude 1 µPa located at the point of coordinates x = 0.0, y = 0.0 and z = 0.3.

The model is axially symmetric. The decomposition in elementary problems with respect to the xOy plane is not operational.

The mesh is limited to the surface generatrix. It is shown in Fig. III.9.

Figure III.9: Mesh used in example case 2

III.3 Header

The header contains comments that describe the problem. The header is fully copied in the first page the result file. The first line of the header is copied at the beginning of each page of the result file. All lines of the header must begin with a star “*”.

```
* Line 1
* Line 2
...
* Line n
```

Example case 1:

```
* CYLINDER WITH QUASI-ELLIPTICAL CROSS SECTION
```

Example case 2:

```
* CYLINDER WITH HEMISPHERICAL ENDCAPS
```

III.4 List of Commands

The commands accepted by **EQI** are listed alphabetically and explained in this section.

The formatting rules for the **EQI** data file are as follows:

- Control characters

"**b**" or "**,**"	equivalent to a space; used to separate two words in the same command line
"/" or "**=**"	equivalent to a line break when inserted in a command line
"*"	beginning or end of a comment

- Syntax for real numbers

-1.22E+02	these are valid examples for defining real numbers
-1.22E2	
-1E-3	
-2.	
3	

<u>Example</u>

The following examples have valid syntax and identical meaning.

```
PLANEWAVE
1
1.E-06    90.    45.

PLANEWAVE = 1
1.E-06    90.    45.

PLANEWAVE = 1 / 1.E-06    90.    45.

PLANEWAVE / 1 / 1.E-06    90.    45.
```

All the **EQI** commands are listed in Table III.3. A "**Yes**" in the Mandatory column indicates that the corresponding command must be present in all data files. "**Varies**" means that the presence of the corresponding command may become mandatory in certain cases.

Name	Mandatory	Option or Parameter	Page
ALGORITHM	Yes	**MEMORY**, **SKYLINE** or **BLOCKS**	13
ELEMENTS	Yes	Topology of the mesh elements	14
END	Yes	Indicator of end of data file	16
EXTERNALPOINT	Yes	Coordinates of the test point for the orientation of the elements	17
FARFIELD	No	Description of directivities	18
FREQUENCY	Yes	Value of the frequency	20
GENERATION	Varies	**PRE-ATI**, **POST-ATI** or **POST-ATI TEST**	21
IMPEDANCE	No	**RADIATION**	22
INTEGAXI	Varies	**ELLIPTICAL** or **GAUSSLEGENDRE**	23
INTEGRATION	Yes	Number of Gauss-Legendre points of the Helmholtz zero field integrals	24
INTERPOLATION	Varies	**STORAGE** or **FREQUENCY**	25
MATERIAL	Yes	Characteristics of the surrounding fluid	26
MAXFREQ	Varies	Value of the maximum limit of storage of frequency interpolation	27
NEARFIELD	No	Description of the computation points in the fluid	28
NODES	Yes	Coordinates of the mesh nodes	30
NORMALNF	No	Value of the normalization coefficient of the zero field equations	31
ORIGINS	No	Activation of the multiple origins technique	32
PRESSURE PROGRAMMED	No	Call to a file for the computation of the incident pressures	33
PRINTING	No	Printing level	36
PROBLEM	Yes	**3D**, **AXI** or **OBLICAXI**	37
PROPAGATION	Yes	**RADIATION** or **SCATTERING**	38
RESULTS	No	**CARTESIEN** or **ANGULAR**	41
SELFDIFFRACTION	No	Description of the autodiffraction coefficients	42
SOLVING	Yes	**HELMHOLTZ**, **NULLFILED**, **COUPLING** or **RIGIDSURFACE**	43
SYMMETRY	Varies	Description of the symmetries	45
TIMEDEP	Yes	Time dependence of the variables	47

Table III.3: Available commands for EQI

Note: Only the first 8 characters of a command are used by **EQI**

Warning: In the following sections, options are given with brackets: "**[option 1, option 2, etc.]**." One of these options must follow the command. Other parameters may be used to complete a command, and be used on the line following the command line. Several parameter levels are also possible, and each level is indicated by a line break. Unless otherwise indicated, blank lines must not used to separate various parameters of the same command.

The **DIRECTIVITY** and **FLUID** commands of earlier versions have been replaced with the **FARFIELD** and **NEARFIELD** commands.

III.4.1 ALGORITHM [MEMORY, SKYLINE, BLOCKS]

This command indicates the type of algorithm used. It requires that one of these options be used:

MEMORY	assembly and computation are performed using the computer dynamic memory
SKYLINE	assembly and computation on disk, using an active columns method derived from that of Atila
BLOCKS	assembly by an active columns method or by blocks and solving by blocks, on disk. This option is completed by the following parameters, illustrated in Fig. III.10.

PRIORITY SWAPPING	assembly by active columns on disk
PRIORITY DISKSTORAGE	assembly by blocks on disk

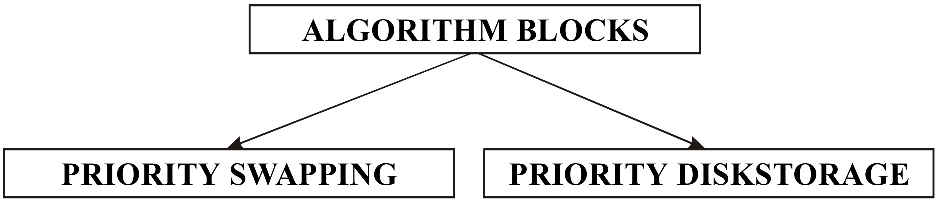


Figure III.10: Parameters completing the BLOCKS option

Note: The **ALGORITHM** command (page 12) must be used before the **RESOLUTION** command.

The **SWAPPING** and **DISKSTORAGE** parameters cannot be cumulated.

Example cases 1 & 2:

```
ALGORITHM MEMORY
```

III.4.2 ELEMENTS

This command indicates the beginning of the mesh element list. The parameters shown in Fig. III.11 complete this command.

INTEG6
n_{11} n_{12} n_{13} n_{14} n_{15} n_{16}
n_{21} n_{22} n_{23} n_{24} n_{25} n_{26}
...
n_{n1} n_{n2} n_{n3} n_{n4} n_{n5} n_{n6}

INTEG8
n_{11} n_{12} n_{13} n_{14} n_{15} n_{16} n_{17} n_{18}
n_{21} n_{22} n_{23} n_{24} n_{25} n_{26} n_{27} n_{28}
...
n_{n1} n_{n2} n_{n3} n_{n4} n_{n5} n_{n6} n_{n7} n_{n8}

INTEG3
n_{11} n_{12} n_{13}
n_{21} n_{22} n_{23}
...
n_{n1} n_{n2} n_{n3}

Figure III.11: Parameters used to complete the ELEMENTS command.

The group of parameters introduced by **INTEG6** (**INTEG8**, respectively) corresponds to the **PROBLEM = 3D** command and indicates the topology of 6-node triangular (8-node quadrilateral) elements where n_{ij} represents the j^{th} node of the i^{th} element, and j takes values between 1 and 6 (1 and 8).

The group of parameters introduced by **INTEG3** corresponds to the **PROBLEM = AXI** and **PROBLEM = OBLICAXI** commands and indicates the topology of 3-node elements where n_{ij} represents the j^{th} node of the i^{th} element, and j takes values between 1 and 3.

Note: The elements are automatically numbered in the order as they appear in the **ELEMENTS** list.

More details about the elements can be found in paragraph III.5.

These groups of parameters can be cumulated at the condition of inserting a blank line between two groups, and making sure that the element types **INTEG6** and **INTEG8** are not present in the same data file than **INTEG3** elements.

Two blank lines are required to terminate the list.

Example case 1:

```
ELEMENTS
INTEG6
*   1 *    7    5   13    6   16   14
*   2 *   20   22   26   21   27   29
... (see paragraph V.2)
*   4 * 158 156 162 157 165 163

INTEG8
*   5 *    5    3   13   11    4   16   15   12
... (see paragraph V.2)
*  48 * 126 124 162 158 125 135 134 163
```

Example case 2:

```
ELEMENTS
INTEG3
*   1 * 29 30 27
*   2 *   8   9   6
... (see paragraph V.2)
* 21 * 21 24 23
```

III.4.3 END

This command indicates the end of the data file.

III.4.4 EXTERNALPOINT = x y z

This command indicates the Cartesian coordinates of an external point used to verify the orientation of the mesh elements. At first, the orientation of the elements is tested, element after element. Then, the program verifies whether the dot product of the normal external to the closest element to the external point with the vector linking the origin of the coordinate system to this point is positive. If not, the whole mesh is inverted. For special geometries, this test may fail.

Example cases 1 & 2:

```
EXTERNALPOINT = 100. 100. 100.
```

III.4.5 FARFIELD PATTERN [BISTATIC, MONOSTATIC]

This command gives rise to the computation of the farfield directivity patterns. The options are:

BISTATIC	computation of the directivity pattern for a radiation load, or a scattering load caused by several one or many incident waves defined in the PROPAGATION command.
MONOSTATIC	computation of the directivity pattern when each of its points is the result of the incident wave of same orientation but opposite direction.

These options are completed by the parameters shown in Fig. III.12.

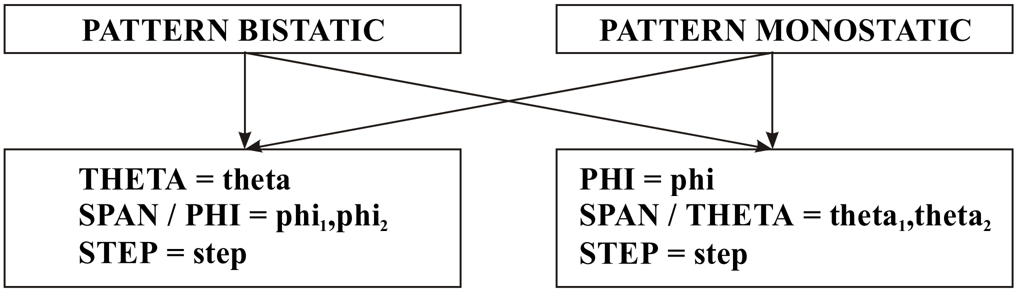


Figure III.12: The group of parameters on the left (on the right, respectively) gives rise to the computation of the directivity pattern in the plane defined by the value of the θ (ϕ) angle, with an angle ϕ (θ) varying by increments in-between the limits $\theta 1$ and $\theta 2$ ($\phi 1$ and $\phi 2$).

Note: These groups of parameters can be cumulated.

A blank line must be used terminate the list.

In the case of a radiation problem, the **BISTATIC** command is mandatory. The directivity patterns requested are generated for each defined load.

The computation of a **MONOSTATIC** directivity pattern requires the use, at least once, of the **PROPAGATION SCATTERING** command in the data file.

Example case 1:

```
FARFIELD
 PATTERN BISTATIC
  THETA = 90.
  SPAN / PHI = 0. 180.
  STEP = 5.
 PATTERN BISTATIC
  PHI = 0.
  SPAN / THETA = 0. 180.
  STEP = 5.
 PATTERN MONOSTATIC
  PHI = 0.
  VARIATION / THETA = 0. 90.
  PAS = 5.
```

Example case 2:

```
FARFIELD
 PATTERN BISTATIC
  PHI = 0.
  SPAN / THETA = 0. 180.
  STEP = 5.
```

III.4.6 FREQUENCY = frequency

This command indicates the value of the excitation frequency. Only one frequency per computation can be defined.

Example case 1:

```
  FREQUENCY = 3000.
```

Example case 2:

```
  FREQUENCY = 1400.
```

III.4.7 GENERATION [PRE-ATI, POST-ATI, POST-ATI TEST]

This command allows generating the matrices necessary to couple to Atila. The options are:

PRE-ATI	computation of the matrix $[A]^{-1}[B]$ and vector $[A]^{-1}\{pi\}$ if needed
POST-ATI	to take into account the displacement field given by Atila and the field resulting from the point source test (see annex III) from which the pressure on Γ is computed
POST-ATI TEST	to take into account the displacement field resulting from the point source test from which the pressure on Γ is computed

Note: The projection of the matrices on the three directions of the Cartesian coordinate system is performed by the **EQIATI** program (see paragraph IV.2).

The **GENERATION** command must be placed before the **PROPAGATION** command.

Example:

```
GENERATION PRE-ATI
```

III.4.8 IMPEDANCE RADIATION REFERENCE

This command gives rise to the computation of the radiation impedance (see annex II). The parameters shown in Fig. III.13 complete this command.

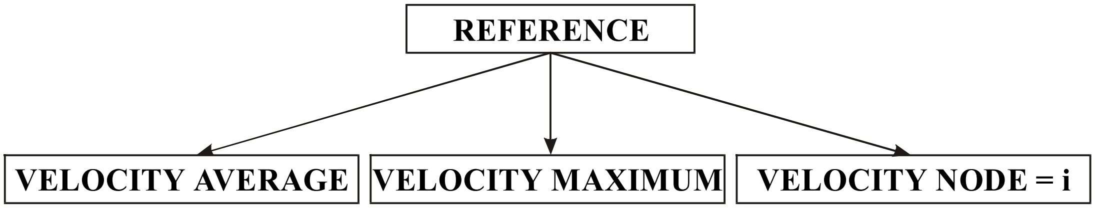


Figure III.13: These parameters indicate the velocity of the reference at the time of computation: the VELOCITY AVERAGE, the VELOCITY MAXIMUM, or the VELOCITY at NODE i.

Note: These parameters cannot be cumulated.

The radiation impedance is computed for each defined radiation load.

Example case 1:

```
IMPEDANCE RADIATION
 REFERENCE / VELOCITY AVERAGE
```

III.4.9 INTEGAXI

This command indicates the use of the Helmholtz integrals method of integration when the **PROBLEM = AXI** or **OBLICAXI** is active. The parameters shown in Fig. III.14 complete this command.

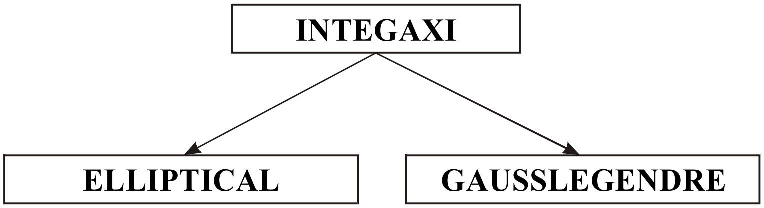


Figure III.14: Parameters to complete the INTEGAXI command.

The **ELLIPTICAL** parameter allows computing the Helmholtz integrals by a semi-analytical method based on the Gauss-Legendre integration along the variable in the generatrix plane and on the use of elliptical integrals in the azimuth angle direction.

The **GAUSSLEGENDRE** parameter gives rise to a Gauss-Legendre integration along the two surface variables.

Note: These parameters cannot be cumulated.

This command can be ignored. In this case, the **ELLIPTICAL** parameter is active.

When the **INTERPOLATION STORAGE** or **PROBLEM OBLICAXI** are used, the **GAUSSLEGENDRE** parameter is mandatory.

III.4.10 INTEGRATION= i_1,i_2,j_1,j_2

This command indicates the number of integration points in the Helmholtz and null field integrals. The integers i_1 and i_2 (j_1 and j_2, respectively) indicate the number of Gauss-Legendre points for each of the two variables of the Helmholtz (null field) integrals. Information about the number of integration points is given in annex V.

Example case 1:

```
INTEGRATION = 2,2,0,0
```

Example case 2:

```
INTEGRATION = 30,3,6,0
```

III.4.11 INTERPOLATION [STORAGE, FREQUENCY]

This command gives rise to the frequency interpolation (see annex IV). The options are:

STORAGE	computation and storage of interpolation matrices for each frequency bound
FREQUENCY	linear interpolation at the working frequency of the interpolation matrices

The parameters shown in Fig. III.15 complete these options.

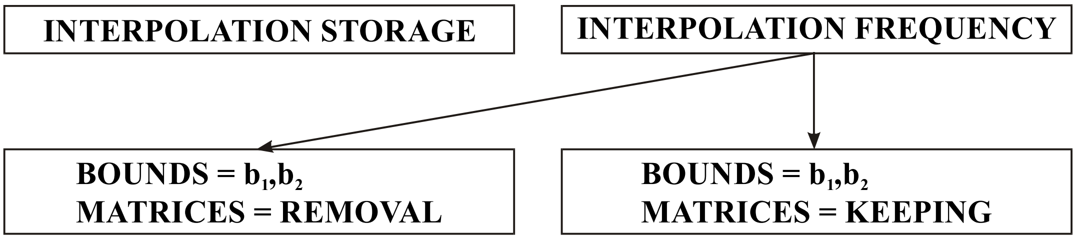


Figure III.15: The frequency interpolation is performed between the interpolation matrices already available at the frequency bounds b_1 and b_2. Once this is done, the user opts for the deletion or storage of the interpolation matrices.

Note: These parameter groups cannot be cumulated.

The **INTERPOLATION** command must be placed before the **PROPAGATION** command.

The projection of the matrices on the three directions of the Cartesian coordinate system is performed by the **EQIAP** program (see paragraph IV.3).

Example:

```
INTERPOLATION FREQUENCY
 BOUNDS = 1000. 3500.
 MATRICES = REMOVAL
```

III.4.12 MATERIAL = name = c_1 c_2

This command indicates the name of the propagation fluid, and its coefficient of isentropic compressibility and density.

Example cases 1 & 2:

```
  MATERIAL = WATER = 0.222E+10 0.1E+04
```

III.4.13 MAXFREQ = frequency

This command indicates the value of the upper bound of the frequency in the case of frequency interpolation (see annex IV). For a given interpolation interval, this value must be known at the time of storage at the bounds and during interpolation at intermediary frequencies.

<u>Example:</u>

```
FREQCMAX = 1500.
```

III.4.14 NEARFIELD

This command generates the computation of the pressure in the fluid at a finite distance from the structure. The parameters shown in Fig. III.16 complete this command.

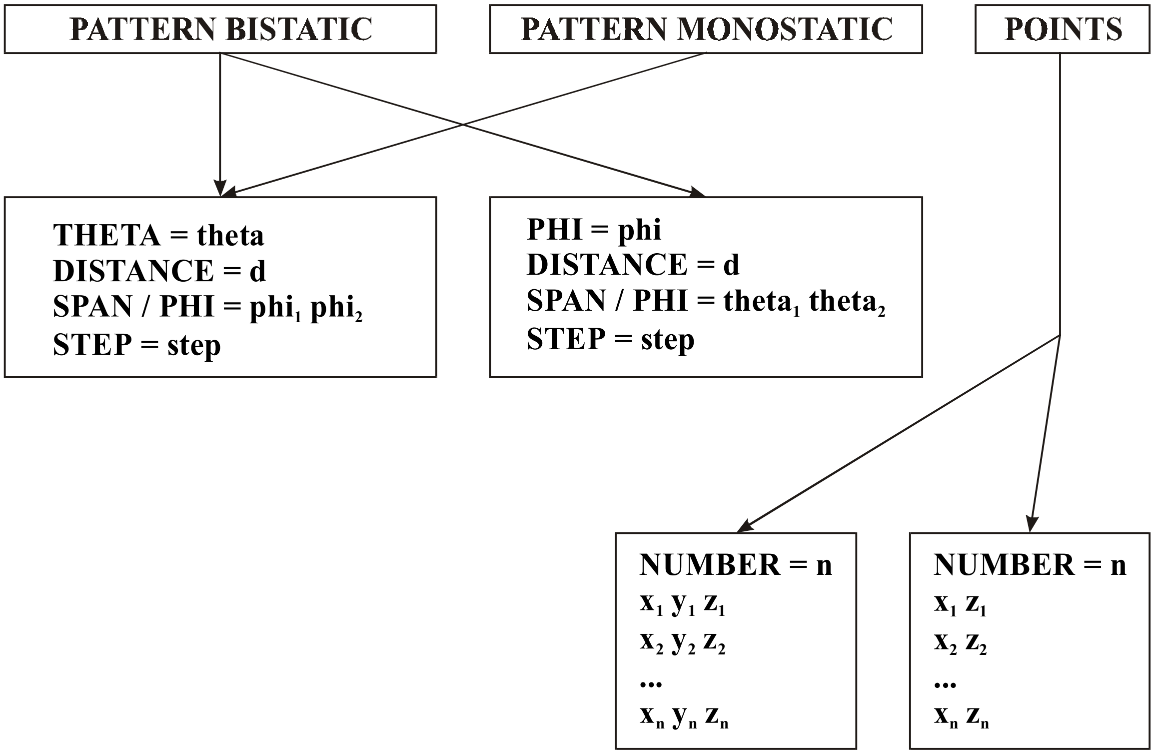


Figure III.16: Parameters used to complete the command.

The **PATTERN BISTATIC** parameter gives rise to the computation of the pressure pattern in the fluid for a radiation load, or a radiation load caused by one or many incident waves defined in the **PROPAGATION** command. The **PATTERN MONOSTATIC** parameter corresponds to the computation of the pressure in the fluid when each point of the latter is the result of the convergent incident wave of same orientation by opposite direction. In both cases, the pattern points are located in the plane defined by the value of the θ or ϕ angle, at a distance **d** taken from the origin if the Cartesian coordinate system, and making an angle ϕ or **q** varying by increment between the limits indicated.

The **POINTS** parameters gives rise to the computation of the pressure in the fluid at a number n of points defined by three (two, respectively) Cartesian coordinates if **PROBLEM = 3D** (**PROBLEM = AXI** or **PROBLEM = OBLICAXI**).

Note: These parameter groups can be cumulated.

A blank line is required to terminate the list.

In the case of a radiation problem, the **NEARFIELD** command cannot be followed with the **PATTERN MONOSTATIC** parameter. The requested diagrams and points are computed for each defined load.

The computation of a **MONOSTATIC** directivity pattern requires the use, at least once, of the **PROPAGATION SCATTERING** command in the data file.

The **POINTS** command can be used only once (one pattern contains all points defined).

The **NEARFIELD** command must be placed after the **ALGORITHM** command.

Example case 2:

```
NEARFIELD
 PATTERN BISTATIC
  PHI = 0.
  DISTANCE = 5.
  SPAN / THETA = 0. 180.
   STEP = 5.
```

III.4.15 NODES

This command initiates the list of the Cartesian coordinates x_i, y_i, and z_i of the mesh nodes, according to Fig. III.17.

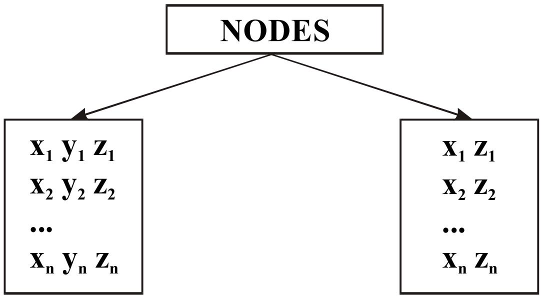


Figure III.17: The parameter group to the left corresponds to the PROBLEM=3D command. The parameter group to the right corresponds to the PROBLEM=AXI or PROBLEM=OBLICAXI.

Note: The nodes are numbered in the order as they appear in the list.

The list of nodes for example cases 1 & 2 can be found in paragraph V.2.

A blank line must be used to terminate the list.

Example case 1:

```
NODES
*   1 *  0.0000E+00   0.0000E+00   0.2870E+00
*   2 * -0.0484E+00   0.0000E+00   0.2870E+00
... (see paragraph V.2)
* 165 *  0.1900E+00   0.0348E+00  -0.2870E+00
```

Example case 2:

```
NODES
*   1 *  2.58800E-01  -1.96600E+00
*   2 *  5.00000E-01  -1.86600E+00
... (see paragraph V.2)
* 42 *  1.00000E+00   8.88900E-01
```

III.4.16 NORMALNF = coefficient

This command indicates the value of the normalization coefficient in the null field equations.

Note: This command can be ignored. In this case, the coefficient is set to 1.

Example:

```
  NORMALCN = 500.
```

III.4.17 ORIGINS

This command activates the multiple origins technique defined by their number n and their Cartesian coordinates x_i, y_i, and z_i. Then, the activation of the close nodes is defined, as shown in Fig. III.18.

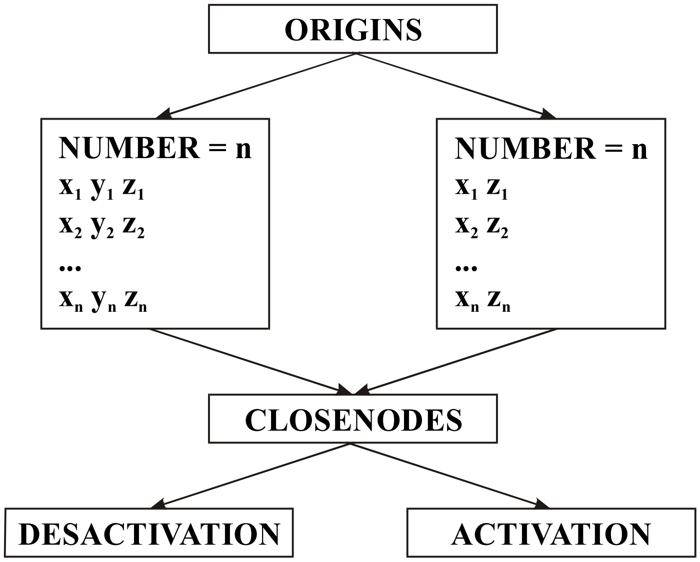


Figure III.18: The parameter group to the left corresponds to the PROBLEM = 3D command. The parameter group to the right corresponds to the PROBLEM = AXI and PROBLEM = OBLICAXI commands.

<u>Note:</u> This command can be ignored. In this case, the origin of the null field equations is the origin of the coordinate system.

<u>Example:</u>

```
ORIGINS
NUMBER = 3
0. -1.
0. 0.
0. 1.
CLOSENODES = ACTIVATION
```

III.4.18 PRESSURE PROGRAMMED

This command activates the call to a user’s subroutine that is used to define the incident pressure. This function can be installed using a shared library.

Note: This command can be ignored. In this case, the loads defined in the data file are used.

The default listing of the proposed function, called **INPREQ**, is given below.

```
FUNCTION INPREQ(X,Y,Z,K,ILOAD,IPARAM)
******PROGRAMMED BY R.BOSSUT ON: 99/06/28
*-------------------------------------------------------------------
* FUNCTION:
*    Compute the incident pressure to point (X,Y,Z) for the wave
*    number K, for the load # ILOAD, whose parameter is IPARAM.
*    Here, we simply compute exp(j*k*R):
*    k is the wave number
*    R is the position vector considered (X,Y,Z)
*
* INPUT VARIABLES:
*    X,Y,Z  (R*8): Point coordinates
*    K      (R*8): Magnitude of the wave vector
*    ILOAD  (I*4): # of load considered
*    IPARAM (I*4): parameter from the PRESSURE PROGRAMMED command
*
* OUTPUT VARIABLE:
*    INPREQ (C*16) : Pressure at the point.
*
* SUBROUTINE :
*   WARNING: IT IS NOT NECESSARY TO USE COMMONS IF THIS FUNCTION IS
*              STORED IN A SHARED LIBRARY BECAUE THEY ARE STORED AT
*              OTHER ADDRESSES THAN COMMONS OF PROGRAMS HAVING THE
*              SAME NAME.
*
*              IF YOU ABSOLUTELY WANT TO USE COMMONS, WE CAN
*              DO THE FOLLOWING ON HP-UX
```

```
*              1) Compile the code with +z -S options instead of -c.
*                 This generates an file in assembly language
*                 (code.s).
*              2) Edit the assembly file, and for each common,
*                  transform the line:
*                     mycommon  .COMM length
*                  into:
*                     .IMPORT mycommon,DATA
*               3) Compile the assembly: cc -c code.s
*
*               OTHERWISE, WRITE THE CODE IN C AND DECLARE THE
*               COMMONS USING: extern struct { ... } mycommon;
*-------------------------------------------------------------------
*
      DOUBLE PRECISION K,X,Y,Z
      COMPLEX*16 INPREQ
*
      INCLUDE "../inc/LUNIT"
*
      DOUBLE PRECISION DP
      LOGICAL FIRST
      DATA FIRST /.TRUE./
*
* Case of an incident plane wave with axial symmetry and no symmetry
* plane at X=0, going from right to left.
*
      DP = -K*X
*
      IF (FIRST) THEN
        WRITE (LUOUT,*) ' '
        WRITE (LUOUT,*) 'Default INPREQ subroutine called :'
        WRITE (LUOUT,*) 'Plane along negative Ox direction.'
        WRITE (LUOUT,*) ' '
      ENDIF
*
```

```
* Time dependence here : exp(-iwt)
*
      INPREQ = DCMPLX(DCOS(DP),DSIN(DP))
*
      FIRST = .FALSE.
      RETURN
      END
```

III.4.19 PRINTING = level

This command indicates the printing level, between 0 and 4, of the result file. By default, the printing level is minimum and is set to zero. This is sufficient for common use of the program. Higher printing levels are usually used for development purposes. The amount of information returned in the result file is defined by the following levels:

1 & higher	directivity patterns or pressures in the fluid at a finite distance from the structure in the case of the point source test
2 & higher	nodal pressures in the case of the point source test, values of the 4π-normed solid angles at the nodes, and topology of the elements immediately after they are read from the data file
3 & higher	coefficients of the pressure decomposition into double Fourier series in the null field equations
4 & higher	coefficients of the final linear system to be solved

Note: This command can be ignored. In this case, the printing level is set to zero.

Example:

```
PRINTING = 2
```

III.4.20 PROBLEM

This command indicates the type of problem to solve. The parameters in Fig. III.19 complete this command.

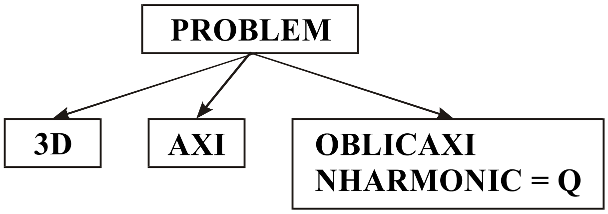


Figure III.19: The 3D parameter corresponds to a tridimensional problem. AXI corresponds to an axially symmetric problem. OBLICAXI corresponds to a problem with partial axial symmetry, limited to scattering computation s and is completed by the number of the harmonic q of the computation defined in paragraph II.5

Note: These parameters cannot be cumulated.

This command can be ignored. In this case, the **3D** parameter is active.

Example case 1:

```
PROBLEM = 3D
```

Example case 2:

```
PROBLEM = AXI
```

III.4.21 PROPAGATION [RADIATION, SCATTERING]

This command indicates the type of load that is causing the propagation. The options are:

RADIATION radiation load

SCATTERING scattering load

This command is completed by the parameters shown in Fig. III.20.

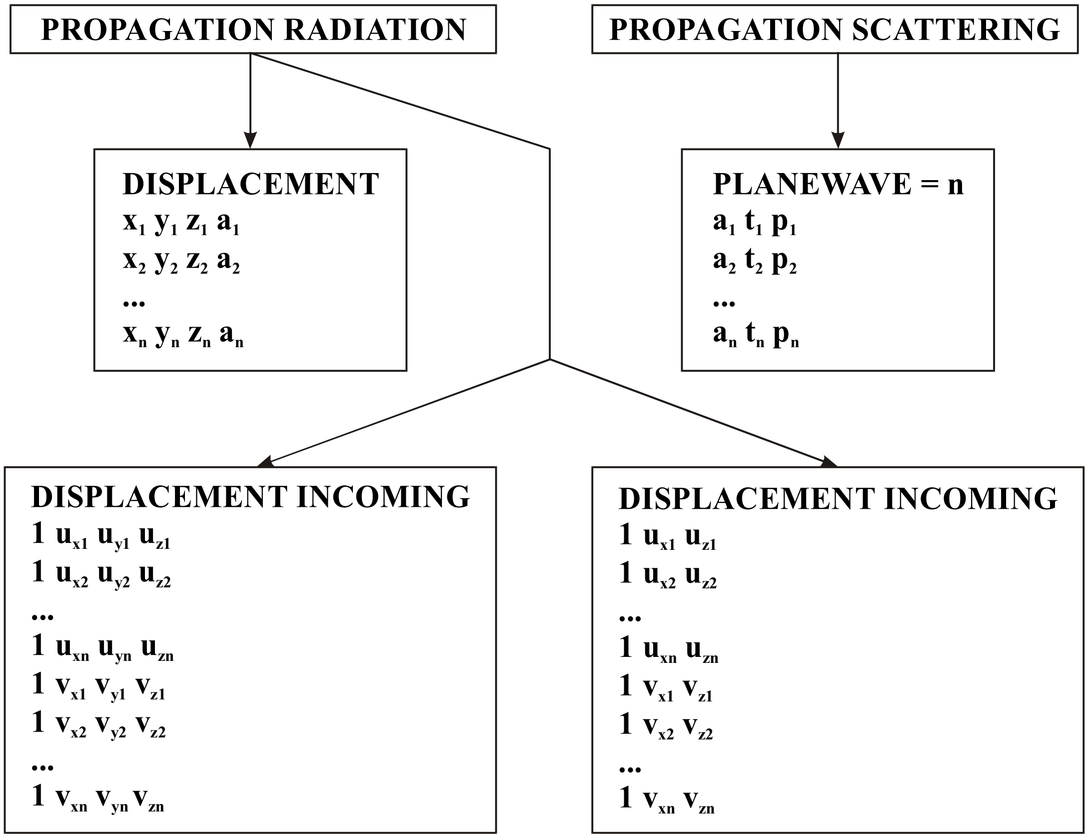


Figure III.20: Parameters for the PROPAGATION command.

The group of parameters introduced by **DISPLACEMENT ANALYTICAL** gives rise to the point source (see annex III). X_i, y_i, and z_i are the Cartesian coordinates of the i^{th} radiating source of amplitude a_i, and enclosed in the structure.

The group of parameters introduced by **DISPLACEMENT EXTERNAL** gives rise to the computation of the radiation problem when the displacement field is imposed. The first part contains the node number i and the real part of the displacement in the three (two, respectively) directions, i.e. u_{xi}, u_{yi}, and u_{zi} (u_{xi} and u_{zi}), when **PROBLEM = 3D** (**PROBLEM = AXI** and **PROBLEM = OBLICAXI**). The displacement must be given for all nodes of the mesh. The order in which the nodal displacements are defined is arbitrary.

The group of parameters introduced by **PLANEWAVE = n** gives rise to the computation of the scattering problem with the structure assumed to be perfectly rigid, submitted simultaneously to n incident waves. Each wave has an amplitude a_i, and assuming that the origin of the orientation vector is the same than the origin of the coordinate system, is carried by the directions θ_0 and φ_0 with the values t_i and p_i.

Note: These groups of parameters can be cumulated.

A blank line must terminate the list.

If the **PROPAGATION SCATTERING** command is followed by **PLANEWAVE** and by the definition of incidence, or if the **PROPAGATION RADIATION** command without any parameters is accompanied by **GENERATION** (**PRE-ATI**, **POST-ATI**, **POST-ATI TEST**), then the vibration of the structure's surface is taken into account through coupling with a finite element program, Atila in particular. In this case, the only other load that can be taken into account in the same computation is the one generated by the point source test via the **PROPAGATION RADIATION** command followed by **DISPLACEMENT ANALYTICAL** and by the definition of the point sources.

The simultaneous call to the **PROPAGATION SCATTERING** command followed by **PLANEWAVE** and by the definition of the incidence and the **PROPAGATION RADIATION** command without parameters is forbidden.

In presence of the **PROPAGATION SCATTERING** command, the definition of plane waves must be removed when the patterns requested are defined by the **PATTERN MONOSTATIC** option.

Example case 1:

```
PROPAGATION SCATTERING
 PLANEWAVE = 1 / 1.E-06 90. 0.

PROPAGATION RADIATION
 DISPLACEMENT ANALYTICAL
  0.05 0.0 ° 0.1 1.E-06
```

Example case 2:

```
PROPAGATION SCATTERING
 PLANEWAVE = 1 / 1.E-06 180. 0.

PROPAGATION RADIATION
 DISPLACEMENT INCOMING
  1    0.   0.   0.
  2    0.   0.   0.
... (see paragraph V.2)
 43  1.E-9  0.   0.
  1    0.   0.   0.
  2    0.   0.   0.
... (see paragraph V.2)
 43    0.   0.   0.
```

III.4.22 RESULTS

This command allows to select the type of result presentation. The parameters in Fig. III.21 complete this command.

Figure III.21: In the case of the CARTESIAN parameter, the pressures are decomposed into real and imaginary parts. In the case of the ANGULAR parameter, the pressures are decomposed into magnitude and phase.

<u>Example cases 1 & 2:</u>

```
RESULTS = ANGULAR
```

III.4.23 SELFDIFFRACTION

PAIR = r_1,d_1 / r2,d_2 / ... / r_n,d_n

This command gives rise to the computation of the n coefficients of selfdiffraction. The integers r_i and d_i represent the number in which the radiation and scattering loads appear in the data file. The definition of the selfdiffraction coefficient is recalled in annex II.

Note: A blank line is required to terminate the list.

Example:

```
SELFDIFFRACTION
PAIR = 2,1
```

III.4.24 SOLVING [HELMHOLTZ, NULLFIELD, COUPLING, RIGIDBAFFLE]

This command indicates the type of computation model. The options are:

HELMHOLTZ	external Helmholtz integrals representation
NULLFIELD	null field method
COUPLING	combination of **HELMHOLTZ** and **NULLFIELD**
RIGIDBAFFLE	radiation of a piston in a rigid baffle

The parameters shown in Fig. III.22 complete this command.

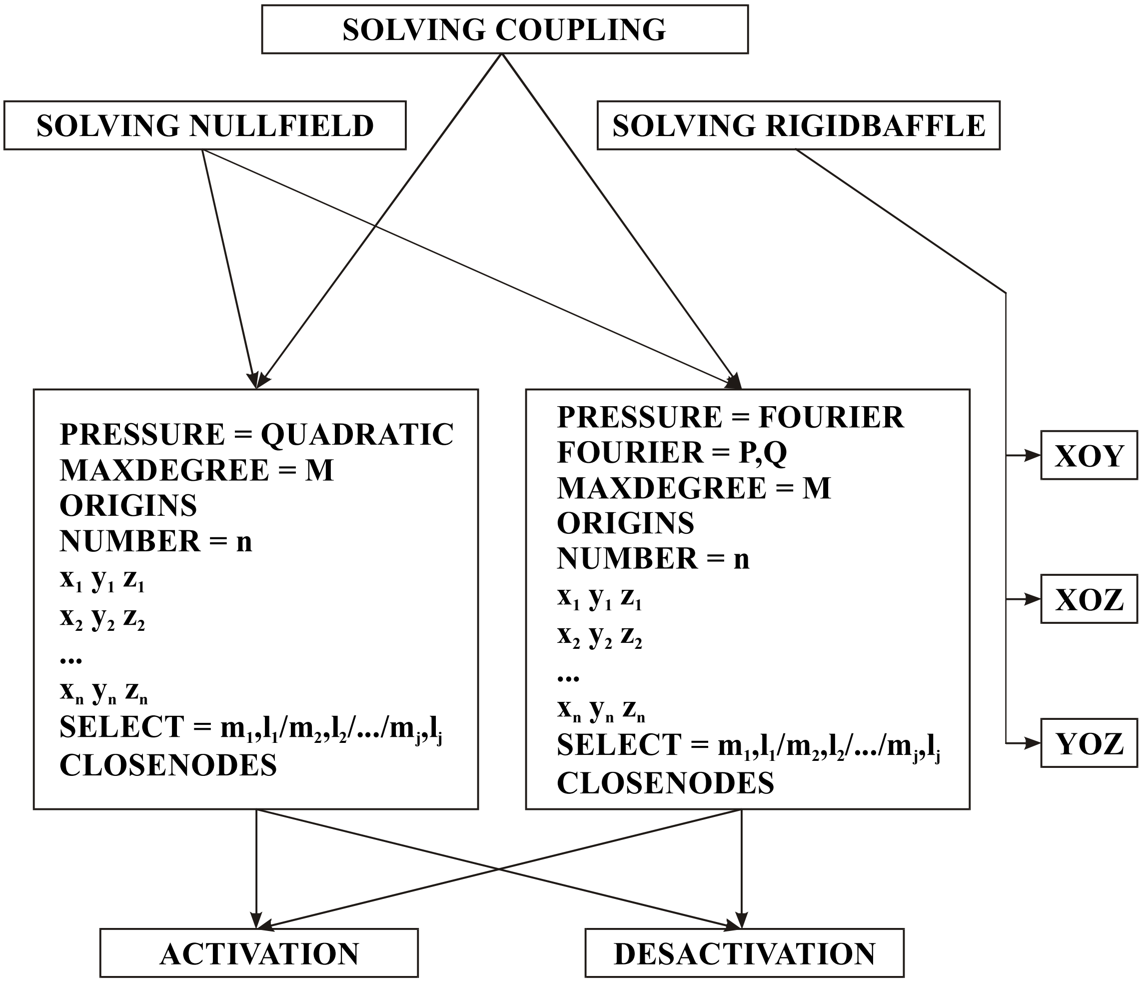


Figure III.22: Parameters that complete the SOLVING command.

The two parameter groups dealing with the null field method and its combination with the external Helmholtz integrals representation begin with the choice of the approximation of the pressure in the null field equations: the classical quadratic approximation or the double Fourier series decomposition. In the latter case, the number of coefficients for the decomposition is set by P and Q. The maximum degree of the null field equations is given by M. These equations are computed with respect to n origins located in the structure having Cartesian coordinates x_i, y_i, and z_i. It is possible, but optional, to select the null field equations of indices m and l. In this latter case, a blank line must terminate the enumeration. The integers P, Q, M, m and l are defined in paragraph II.1.

When the close nodes technique is activated, the null field equations are computed at each node only with respect to the closest origin.

The parameters of the rigid baffle determine the reference plane of the baffle.

Note: These parameter groups cannot be cumulated.

The parameter group introducing the multiple origins can be ignored. In this case, the computation is performed with only one origin merged with the origin of the mesh nodes coordinates.

The **SOLVING** command must be placed before the **ALGORITHM** command.

If the **SOLVING NULLFIELD** command is activated, then there must be as many null field equations as unknowns, those being the nodal pressures for **the PRESSURE = QUADRATIC** parameter or the Fourier coefficients for the **PRESSURE = FOURIER** parameter.

Example case 1:

```
SOLVING HELMHOLTZ
```

Example case 2:

```
SOLVING COUPLING
 PRESSURE = FOURIER
 FOURIER = 20,0
 MAXDEGREE = 1
 ORIGINS
 NUMBER = 3
  0. 0. 1.
  0. 0. 0.
  0. 0. -1.
 CLOSENODES = DESACTIVATION
```

III.4.25 SYMMETRY

This command indicates the problem symmetries with respect to the orthogonal planes formed by the axes of the Cartesian coordinate system. The parameters shown in Fig. III.23 complete this command.

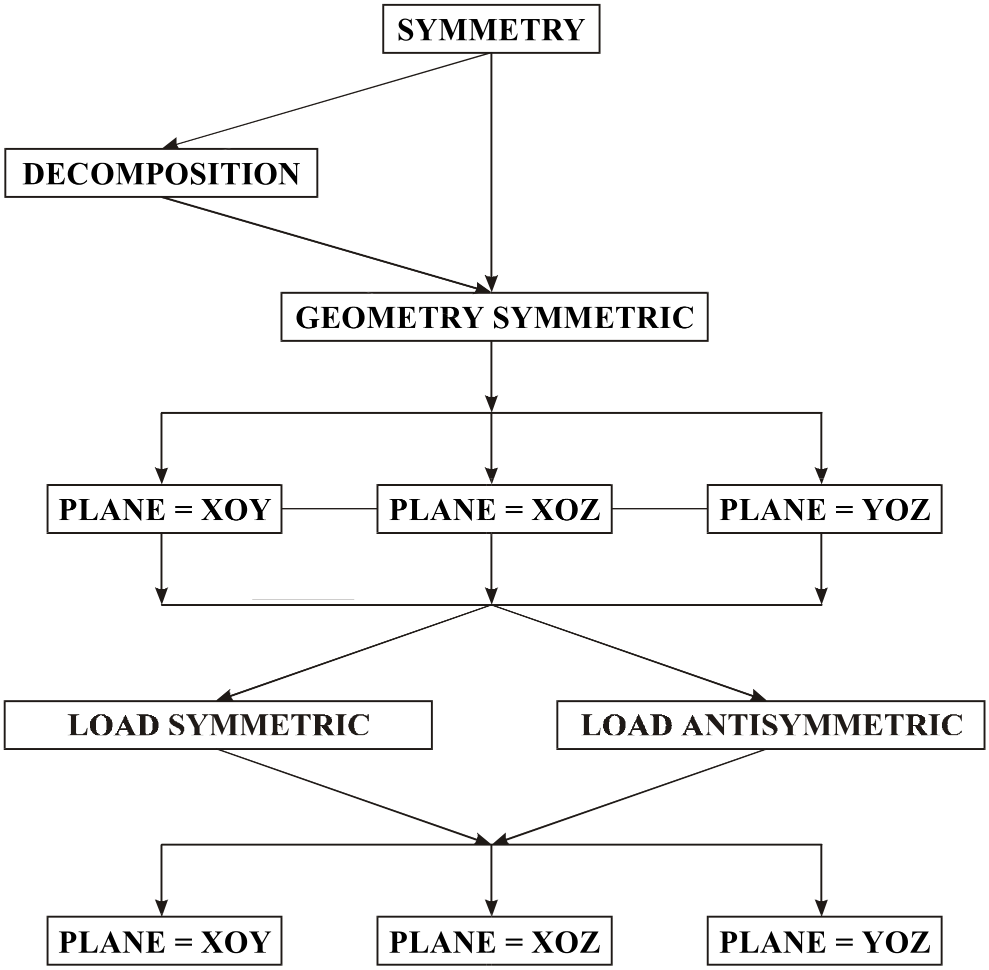


Figure III.23: The indication of the decomposition in elementary problems before defining the problem symmetries, is optional. Then, one, two, or three symmetry planes of the geometry are defined. If needed, planes of symmetry or antisymmetry of the load are indicated.

Note: The **SYMMETRY** command is not operational when **PROBLEM = AXI** or **PROBLEM = OBLICAXI**.

The three symmetry planes must contain the origin of the Cartesian coordinate system, and, if needed, the origins of the null field equations.

A symmetry or antisymmetry of the load is necessarily a symmetry of the geometry. The reciprocal is false.

Several loads can be taken into account during one computation. The symmetries or antisymmetries defined must therefore correspond to the most unfavorable load case.

The existence of a symmetry or antisymmetry plane of the load allows reducing the mesh to the part located in the positive half-space. Conversely, a symmetry plane of the geometry (without symmetry or antisymmetry of the load) does not allow this mesh reduction. In this latter case, the boundary line defined by the intersection of this plane with the structure must be composed of surface element sides (no elements crossing that line).

When the definition of parameters allows it (the number of symmetry planes of the geometry is larger than the plane of symmetries or antisymmetries of the load), the **DECOMPOSITION** option is recommended: the decomposition in elementary problems allows decreasing the time and memory required for the computation.

Example case 1:

```
SYMMETRY
 GEOMETRY SYMMETRIC
  PLANE = XOY
  PLANE = XOZ
  PLANE = YOZ
 LOAD SYMMETRIC
  PLAN = XOZ
```

III.4.26 TIMEDEP

This command indicates the time dependency of the variables in $e^{-i\omega t}$ or $e^{+i\omega t}$. The parameters are shown in Fig. III.24.

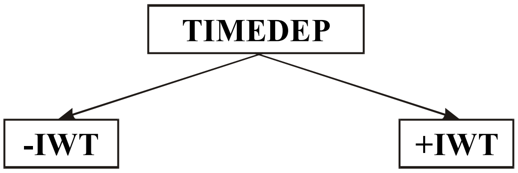


Figure III.24: The –IWT parameter corresponds to $e^{-i\omega t}$ and +IWT to $e^{+i\omega t}$.

Note: These parameters cannot be cumulated.

Example cases 1 & 2:

```
  TIMEDEP = -IWT
```

III.5 List of Available Elements

The precision of computation results depends in part on the type of elements constituting the mesh. Before constructing the mesh, it is therefore necessary to know the available elements and their domain of validity. Elements accepted by EQI are isoparametric with quadratic variation. They allow a good approximation of curved surfaces with a small number of elements, and consequently, of nodes.

III.5.1 Case of a Tri-Dimensional Problem

This is the model that corresponds to the PROBLEM = 3D command. Two elements are available, and shown in Fig. III.25 and III.26. The faces represented in these figures correspond to the external surface of the mesh. For these two elements, the entry parameters are:

INTEG6	**INTEG8**
$\mathbf{N_1\ N_2\ N_3\ N_4\ N_5\ N_6}$	$\mathbf{N_1\ N_2\ N_3\ N_4\ N_5\ N_6\ N_7\ N_8}$

where N_1 to N_6 (to N_8, respectively) represent the mesh node numbers according to the topology shown in Fig. III.25 and III.26. Respecting this topology is essential.

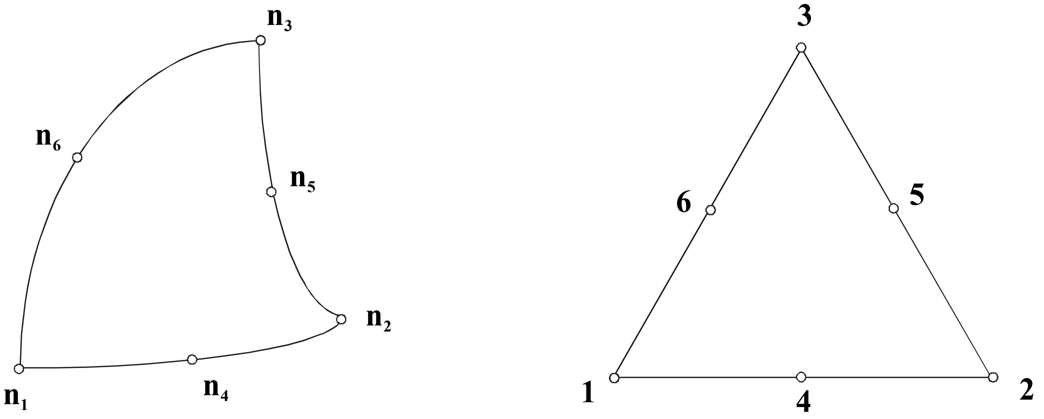


Figure III.25: 6-node triangular element

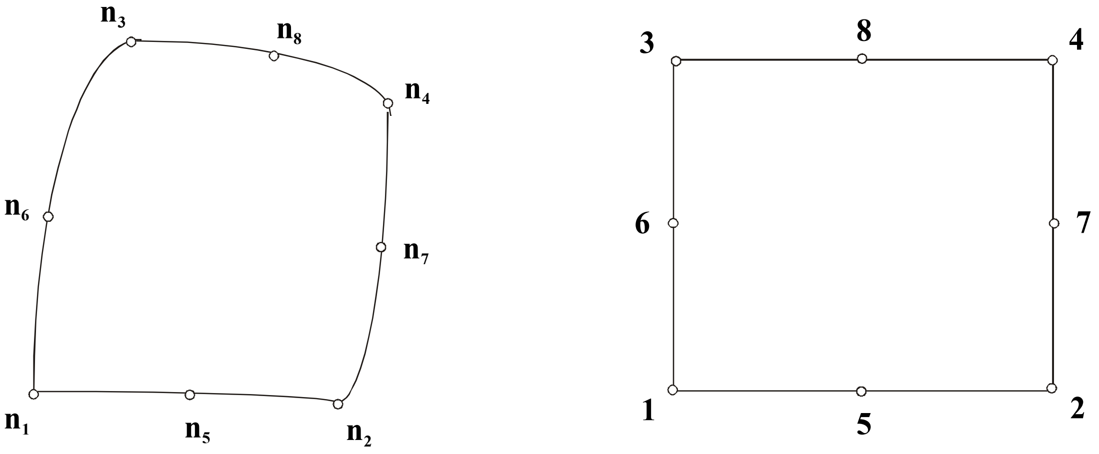


Figure III.26: 8-node quadrilateral element

III.5.2 Case of a Problem with Complete or Partial Axial Symmetry

This is the model that correspond to the PROBLEM = AXI and PROBLEM = OBLICAXI commands. The only available element is shown in Fig. III.27. The positive direction for this element is that of the generatrix taken from its highest point to its lowest. The entry parameters are:

INTEG3

N_1 N_2 N_3

where N_1 to N_3 represent the mesh node numbers according to the topology shown in Fig. III.27. Respecting this topology is essential.

Figure III.27: 3-node linear element

III.5.3 Element Validity

To guarantee a good numerical behavior of the mesh, some simple rules must be respected:

- *λ/4 criterion*: the longest dimension of each element must be less than a quarter of the wavelength.
- *Eccentricity ratio*: in the case of tridimensional problems, the ratio between the longest and the smallest dimensions of each element must be less than 3.

- *Angle constraints*: in the case of tridimensional problems, the angle between two adjacent sides of a triangle (a quadrilateral, respectively) must be within 30º and 100º (within 45º and 135º).
- *Minimum curvature radius*: in the case of tridimensional problems, the curvature radius of an element must be larger than its longest dimension.
- *Coherence between nodes*: in the case of tridimensional problems, a node at the vertex (middle of the side, respectively) of an element cannot be the middle of the side (vertex) of another element. Moreover, two elements cannot overlap.
- *Direction of element topology*: this direction determines the orientation of the normal to the surface which, by convention, is taken positive.
- *Surface closure*: symmetries excepted, the structure's surface must be closed.

EQI

ISEN-LILLE

Chapter IV: Associated Programs

IV.1 EQILG

The **EQILG** program was developed to fulfill two functions:

- Test the coherence of the **EQI** data file, i.e. verify the orientation of the mesh elements and the compatibility of the commands
- Compute the maximum frequency that is compatible with the $\lambda/4$ criterion

The results file generated after running EQILG contains the commands the program reads, as well as their attributes and/or parameters. Incompatibilities between commands are indicated. Then, the frequency corresponding to the largest dimension of each element is indicated. Finally, the largest of these frequencies is the maximum frequency that can be used with the mesh.

Note: The coherence tests of the data file do not formally guarantee its validity.

IV.2 EQIATI

The **EQIATI** program is used to model a fluid-structure interaction problem. The principle of the coupling with **ATILA** can be decomposed into four steps. At the end of each step, the matrices and/or vectors obtained are stored into separate files, which are then available on the hard disk. These steps and files obtained are described below.

IV.2.1 First Step: Running EQI

- Computation of the $[A]^{-1}$ matrix and of the data required for the assembly of the [B] matrix
- Computation of the $\{p_i\}$ vector

File	**Name**
$[A]^{-1}$	MATA
[B]	MATB
$\{p_i\}$ Bistatic case	PRESB
$\{p_i\}$ Monostatic case	PRESM

Table IV.1: Files generated after running EQI (first step of a fluid-structure problem).

IV.2.2 Second Step : Running EQIATI to Interface EQI and ATILA

- Computation of the $[B_x]$, $[B_y]$ and $[B_z]$ matrices
- Computation of the $[A]^{-1}[B_x]$, $[A]^{-1}[B_y]$ and $[A]^{-1}[B_z]$ matrices
- Computation of the $[A]^{-1}\{p_i\}$ vector

File	Name
$[A]^{-1}[B_x]$	MATBX
$[A]^{-1}[B_y]$	MATBY
$[A]^{-1}[B_z]$	MATBZ
$[A]^{-1}\{p_i\}$ Bistatic case	PRESB
$[A]^{-1}\{p_i\}$ Monostatic case	PRESM

Table IV.2: Files generated after running EQIATI (second step of a fluid-structure problem).

IV.2.3 Third Step : Running ATILA

- Computation of the nodal displacement vector for the volume mesh
- Extraction of the $\{u_x\}$, $\{u_y\}$ and $\{u_z\}$ vectors

File	Name
$\{u_x\}$	VECUX
$\{u_y\}$	VECUY
$\{u_z\}$	VECUZ

Table IV.3: Files generated after running ATILA (third step of a fluid-structure problem).

IV.2.4 Fourth Step : Running EQI

- Computation of the {p} vector
- Execution of the post-processing functions

The first and last steps are produced by the **GENERATION PRE-ATI and GENERATION POST-ATI** commands. **The GENERATION POST-ATI TEST** command can be used to suppress the third step by using the displacements obtained by the point source test, and thus validate the numerical behavior of the **EQI** and **EQIATI** programs.

Note: The above technique remains valid after squaring the over-determined system of integral-null-field equations conformably to Paragraph II.4.

Note: The impedance matrices can be used by other finite element programs on the condition of revising the second step.

Note: In the case of a problem with full (**PROBLEM = AXI**) or partial (**PROBLEM = OBLICAXI**) axial symmetry, the projection is done on the axes x and z only.

IV.3 EQIAP

The **EQIAP** program is used when a frequency interpolation of the integral equation matrices is performed. The theoretical formulation is detailed in Appendix IV. The interpolation principle can be decomposed into three steps. At the end of each step, the matrices and/or vectors obtained are stored into separate files, which are then available on the hard disk. These steps and files obtained are described below.

IV.3.1 First Step: Running EQI

- Computation of the $[\hat{A}]$ matrix and of the data required for the assembly of the $[\hat{B}]$ matrix at one of the two frequency bounds of the interpolation interval
- Computation of the $\{\alpha\}$ vector of solid angles

File	Name
$[\hat{A}]$	MATA
$[\hat{B}]$	MATB
$\{\alpha\}$	VECALP

Table IV.4: Files generated after running EQI (first step of a frequency interpolation problem).

IV.3.2 Second Step: Running EQIAP

- Computation of the $[\hat{B}_x]$, $[\hat{B}_y]$ and $[\hat{B}_z]$ matrices at one of the two frequency bounds of the interpolation interval

File	Name
$[\hat{B}_x]$	MATBX
$[\hat{B}_y]$	MATBY
$[\hat{B}_z]$	MATBZ

Table IV.5: Files generated after running EQIAP (second step of a frequency interpolation problem).

IV.3.3 First Step Repeat: Re-Run of the First Step for the Second Frequency Bound

IV.3.4 Second Step Repeat: Re-Run of the Second Step for the Second Frequency Bound

IV.3.5 Third Step: Running EQI

- Computation of the integral equations matrices linearly interpolated between the frequency bounds

The first step is generated by the **INTERPOLATION STORAGE** coupling and the last step by the **INTERPOLATION FREQUENCY** command. After the second step (second step repeat, respectively), it is necessary to rename the files MATA, MATBX, MATBY, and MATBZ of the lower frequency bound (higher frequency bound, respectively) to MATAF1, MATBXF1, MATBYF1, and MATBZF1 (MATAF2, MATBXF2, MATBYF2, and MATBZF2, respectively). The third step can be indefinitely repeated on the condition that the **MATRICES = KEEPING** parameter is selected.

Note: No interpolation is performed on the $\{\alpha\}$ vector since the solid angle is independent of frequency.

Note: In the case of a problem with full (**PROBLEM = AXI**) or partial (**PROBLEM = OBLICAXI**) axial symmetry, the projection is done on the axes x and z only.

IV.4 OBLIC

The **OBLIC** program does the post-processing of the partial axial symmetry model, which is dedicated to the computation of the scattering by an axially symmetric target under oblique incidence. This program allows the generation of the pressure and displacement fields in the whole space, conformably to equations (24) and (25) of Paragraph II.5.

OBLIC's operation principle can be decomposed into two steps: first, the fields in the xOz reference plane are computed for every cylindrical harmonic; second, these fields are computed for the whole space via their expansion in Fourier series.

Two possibilities are offered: the first refers to the scattering by a rigid target under oblique incidence; the second refers to the scattering of an elastic structure under oblique incidence. In this last case, the fluid-structure coupling is performed using **ATILA**, as described in Paragraph IV.2.

IV.4.1 Scattering by a rigid target

IV.4.1.1 Loop on the Harmonics – First Step: Running EQI for the Harmonic q Considered in the xOz Plane

- Computation of the incident pressure p_i^q, the surface pressure p^q, the pressure in the fluid at finite distance p_f^q, the far-field pressure in the fluid p_∞^q, and, in the case of the point-source test, the radial displacements u_ρ^q along the Ox axis and u_z^q along the Oz axis.

File	Name
p_i^q	CHAMP0
u_ρ^q and u_z^q	CHAMP1
p^q	CHAMP2
p_f^q	CHAMP3
p_∞^q	CHAMP4
Problem parameters	DEFI

IV.4.1.2 End of Loop – Second Step: Running OBLIC

- Summation on the harmonics and computation of the fields in the whole space

IV.4.2 Scattering by an Elastic Target

IV.4.2.1 Loop on the Harmonics – First Step: Fluid-Structure Coupling for the Harmonic q in the xOz Plane (see Paragraph IV.2)

- First step: Running the **EQI** program
- Second step: Running the **EQIATI** program to interface **EQI** and **ATILA**
- Third step: Running the **ATILA** program
- Fourth step: Running the **EQI** program

IV.4.2.2 End of Loop – Second Step: Running OBLIC

- Summation on the harmonics and computation of the fields in the whole space

EQI

ISEN-LILLE

Chapter V: EXAMPLES

V.1 Introduction

This chapter contains the data and result files for the two example cases presented in section III.2. Several parts of the results file are commented.

According to the description of the **PRINTING** command in paragraph III.4, the **PRINTING** level is set to zero by default, and the nodal pressures, directivity factor and pressures in the fluid are not given in the results file for the point source test. A simple summary of the error computation on the nodal pressures is given.

The directivity patterns (fluid pressures, respectively) are grouped in a special file with the **PAT** extension (PATFL), and can be used, for instance, by the **PATTERN** program of **Atila**.

V.2 Data Files for the Example Cases

V.2.1 Data File for Example Case 1

The content of the `case1.eqi` file is fully given below. Please refer to Chapter III for more details about the commands used in this file.

```
* CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
NODES
*   1 *  0.0000E+00  0.0000E+00  0.2870E+00
*   2 * -0.0484E+00  0.0000E+00  0.2870E+00
*   3 * -0.1054E+00  0.0000E+00  0.2870E+00
*   4 * -0.1441E+00  0.0000E+00  0.2870E+00
*   5 * -0.1900E+00  0.0000E+00  0.2870E+00
*   6 * -0.2161E+00  0.0000E+00  0.2870E+00
*   7 * -0.2420E+00  0.0000E+00  0.2870E+00
*   8 *  0.0000E+00  0.0528E+00  0.2870E+00
*   9 *  0.0000E+00  0.1050E+00  0.2870E+00
*  10 * -0.0484E+00  0.1043E+00  0.2870E+00
*  11 * -0.1054E+00  0.0946E+00  0.2870E+00
*  12 * -0.1441E+00  0.0863E+00  0.2870E+00
*  13 * -0.1900E+00  0.7047E-01  0.2870E+00
*  14 * -0.2273E+00  0.0428E+00  0.2870E+00
*  15 * -0.1054E+00  0.0484E+00  0.2870E+00
*  16 * -0.1900E+00  0.0348E+00  0.2870E+00
*  17 *  0.0484E+00  0.0000E+00  0.2870E+00
*  18 *  0.1054E+00  0.0000E+00  0.2870E+00
*  19 *  0.1441E+00  0.0000E+00  0.2870E+00
*  20 *  0.1900E+00  0.0000E+00  0.2870E+00
*  21 *  0.2161E+00  0.0000E+00  0.2870E+00
*  22 *  0.2420E+00  0.0000E+00  0.2870E+00
*  23 *  0.0484E+00  0.1043E+00  0.2870E+00
*  24 *  0.1054E+00  0.0946E+00  0.2870E+00
*  25 *  0.1441E+00  0.0863E+00  0.2870E+00
*  26 *  0.1900E+00  0.7047E-01  0.2870E+00
*  27 *  0.2273E+00  0.0428E+00  0.2870E+00
*  28 *  0.1054E+00  0.0484E+00  0.2870E+00
*  29 *  0.1900E+00  0.0348E+00  0.2870E+00
*  30 *  0.0000E+00  0.1050E+00  0.2645E+00
*  31 * -0.1054E+00  0.0946E+00  0.2645E+00
*  32 * -0.1900E+00  0.7047E-01  0.2645E+00
*  33 * -0.2420E+00  0.0000E+00  0.2645E+00
*  34 *  0.2420E+00  0.0000E+00  0.2645E+00
*  35 *  0.1900E+00  0.7047E-01  0.2645E+00
*  36 *  0.1054E+00  0.0946E+00  0.2645E+00
*  37 *  0.0000E+00  0.1050E+00  0.2400E+00
*  38 * -0.0484E+00  0.1043E+00  0.2400E+00
*  39 * -0.1054E+00  0.0946E+00  0.2400E+00
*  40 * -0.1441E+00  0.0863E+00  0.2400E+00
*  41 * -0.1900E+00  0.7047E-01  0.2400E+00
*  42 * -0.2273E+00  0.0428E+00  0.2400E+00
*  43 * -0.2420E+00  0.0000E+00  0.2400E+00
*  44 *  0.2420E+00  0.0000E+00  0.2400E+00
*  45 *  0.2273E+00  0.0428E+00  0.2400E+00
*  46 *  0.1900E+00  0.7047E-01  0.2400E+00
*  47 *  0.1441E+00  0.0863E+00  0.2400E+00
*  48 *  0.1054E+00  0.0946E+00  0.2400E+00
*  49 *  0.0484E+00  0.1043E+00  0.2400E+00
```

```
*   50 *  0.0000E+00  0.1050E+00  0.1815E+00
*   51 * -0.1054E+00  0.0946E+00  0.1815E+00
*   52 * -0.1900E+00  0.7047E-01  0.1815E+00
*   53 * -0.2420E+00  0.0000E+00  0.1815E+00
*   54 *  0.2420E+00  0.0000E+00  0.1815E+00
*   55 *  0.1900E+00  0.7047E-01  0.1815E+00
*   56 *  0.1054E+00  0.0946E+00  0.1815E+00
*   57 *  0.0000E+00  0.1050E+00  0.1210E+00
*   58 * -0.0484E+00  0.1043E+00  0.1210E+00
*   59 * -0.1054E+00  0.0946E+00  0.1210E+00
*   60 * -0.1441E+00  0.0863E+00  0.1210E+00
*   61 * -0.1900E+00  0.7047E-01  0.1210E+00
*   62 * -0.2273E+00  0.0428E+00  0.1210E+00
*   63 * -0.2420E+00  0.0000E+00  0.1210E+00
*   64 *  0.2420E+00  0.0000E+00  0.1210E+00
*   65 *  0.2273E+00  0.0428E+00  0.1210E+00
*   66 *  0.1900E+00  0.7047E-01  0.1210E+00
*   67 *  0.1441E+00  0.0863E+00  0.1210E+00
*   68 *  0.1054E+00  0.0946E+00  0.1210E+00
*   69 *  0.0484E+00  0.1043E+00  0.1210E+00
*   70 *  0.0000E+00  0.1050E+00  0.0605E+00
*   71 * -0.1054E+00  0.0946E+00  0.0605E+00
*   72 * -0.1900E+00  0.7047E-01  0.0605E+00
*   73 * -0.2420E+00  0.0000E+00  0.0605E+00
*   74 *  0.2420E+00  0.0000E+00  0.0605E+00
*   75 *  0.1900E+00  0.7047E-01  0.0605E+00
*   76 *  0.1054E+00  0.0946E+00  0.0605E+00
*   77 *  0.0000E+00  0.1050E+00  0.0000E+00
*   78 * -0.0484E+00  0.1043E+00  0.0000E+00
*   79 * -0.1054E+00  0.0946E+00  0.0000E+00
*   80 * -0.1441E+00  0.0863E+00  0.0000E+00
*   81 * -0.1900E+00  0.7047E-01  0.0000E+00
*   82 * -0.2273E+00  0.0428E+00  0.0000E+00
*   83 * -0.2420E+00  0.0000E+00  0.0000E+00
*   84 *  0.2420E+00  0.0000E+00  0.0000E+00
*   85 *  0.2273E+00  0.0428E+00  0.0000E+00
*   86 *  0.1900E+00  0.7047E-01  0.0000E+00
*   87 *  0.1441E+00  0.0863E+00  0.0000E+00
*   88 *  0.1054E+00  0.0946E+00  0.0000E+00
*   89 *  0.0484E+00  0.1043E+00  0.0000E+00
*   90 *  0.0000E+00  0.1050E+00 -0.0605E+00
*   91 * -0.1054E+00  0.0946E+00 -0.0605E+00
*   92 * -0.1900E+00  0.7047E-01 -0.0605E+00
*   93 * -0.2420E+00  0.0000E+00 -0.0605E+00
*   94 *  0.2420E+00  0.0000E+00 -0.0605E+00
*   95 *  0.1900E+00  0.7047E-01 -0.0605E+00
*   96 *  0.1054E+00  0.0946E+00 -0.0605E+00
*   97 *  0.0000E+00  0.1050E+00 -0.1210E+00
*   98 * -0.0484E+00  0.1043E+00 -0.1210E+00
*   99 * -0.1054E+00  0.0946E+00 -0.1210E+00
*  100 * -0.1441E+00  0.0863E+00 -0.1210E+00
*  101 * -0.1900E+00  0.7047E-01 -0.1210E+00
*  102 * -0.2273E+00  0.0428E+00 -0.1210E+00
*  103 * -0.2420E+00  0.0000E+00 -0.1210E+00
*  104 *  0.2420E+00  0.0000E+00 -0.1210E+00
*  105 *  0.2273E+00  0.0428E+00 -0.1210E+00
*  106 *  0.1900E+00  0.7047E-01 -0.1210E+00
*  107 *  0.1441E+00  0.0863E+00 -0.1210E+00
*  108 *  0.1054E+00  0.0946E+00 -0.1210E+00
*  109 *  0.0484E+00  0.1043E+00 -0.1210E+00
*  110 *  0.0000E+00  0.1050E+00 -0.1815E+00
*  111 * -0.1054E+00  0.0946E+00 -0.1815E+00
*  112 * -0.1900E+00  0.7047E-01 -0.1815E+00
```

```
* 113 * -0.2420E+00  0.0000E+00 -0.1815E+00
* 114 *  0.2420E+00  0.0000E+00 -0.1815E+00
* 115 *  0.1900E+00  0.7047E-01 -0.1815E+00
* 116 *  0.1054E+00  0.0946E+00 -0.1815E+00
* 117 *  0.0000E+00  0.1050E+00 -0.2400E+00
* 118 * -0.0484E+00  0.1043E+00 -0.2400E+00
* 119 * -0.1054E+00  0.0946E+00 -0.2400E+00
* 120 * -0.1441E+00  0.0863E+00 -0.2400E+00
* 121 * -0.1900E+00  0.7047E-01 -0.2400E+00
* 122 * -0.2273E+00  0.0428E+00 -0.2400E+00
* 123 * -0.2420E+00  0.0000E+00 -0.2400E+00
* 124 *  0.2420E+00  0.0000E+00 -0.2400E+00
* 125 *  0.2273E+00  0.0428E+00 -0.2400E+00
* 126 *  0.1900E+00  0.7047E-01 -0.2400E+00
* 127 *  0.1441E+00  0.0863E+00 -0.2400E+00
* 128 *  0.1054E+00  0.0946E+00 -0.2400E+00
* 129 *  0.0484E+00  0.1043E+00 -0.2400E+00
* 130 *  0.0000E+00  0.1050E+00 -0.2645E+00
* 131 * -0.1054E+00  0.0946E+00 -0.2645E+00
* 132 * -0.1900E+00  0.7047E-01 -0.2645E+00
* 133 * -0.2420E+00  0.0000E+00 -0.2645E+00
* 134 *  0.2420E+00  0.0000E+00 -0.2645E+00
* 135 *  0.1900E+00  0.7047E-01 -0.2645E+00
* 136 *  0.1054E+00  0.0946E+00 -0.2645E+00
* 137 *  0.0000E+00  0.0000E+00 -0.2870E+00
* 138 * -0.0484E+00  0.0000E+00 -0.2870E+00
* 139 * -0.1054E+00  0.0000E+00 -0.2870E+00
* 140 * -0.1441E+00  0.0000E+00 -0.2870E+00
* 141 * -0.1900E+00  0.0000E+00 -0.2870E+00
* 142 * -0.2161E+00  0.0000E+00 -0.2870E+00
* 143 * -0.2420E+00  0.0000E+00 -0.2870E+00
* 144 *  0.0000E+00  0.0528E+00 -0.2870E+00
* 145 *  0.0000E+00  0.1050E+00 -0.2870E+00
* 146 * -0.0484E+00  0.1043E+00 -0.2870E+00
* 147 * -0.1054E+00  0.0946E+00 -0.2870E+00
* 148 * -0.1441E+00  0.0863E+00 -0.2870E+00
* 149 * -0.1900E+00  0.7047E-01 -0.2870E+00
* 150 * -0.2273E+00  0.0428E+00 -0.2870E+00
* 151 * -0.1054E+00  0.0484E+00 -0.2870E+00
* 152 * -0.1900E+00  0.0348E+00 -0.2870E+00
* 153 *  0.0484E+00  0.0000E+00 -0.2870E+00
* 154 *  0.1054E+00  0.0000E+00 -0.2870E+00
* 155 *  0.1441E+00  0.0000E+00 -0.2870E+00
* 156 *  0.1900E+00  0.0000E+00 -0.2870E+00
* 157 *  0.2161E+00  0.0000E+00 -0.2870E+00
* 158 *  0.2420E+00  0.0000E+00 -0.2870E+00
* 159 *  0.0484E+00  0.1043E+00 -0.2870E+00
* 160 *  0.1054E+00  0.0946E+00 -0.2870E+00
* 161 *  0.1441E+00  0.0863E+00 -0.2870E+00
* 162 *  0.1900E+00  0.7047E-01 -0.2870E+00
* 163 *  0.2273E+00  0.0428E+00 -0.2870E+00
* 164 *  0.1054E+00  0.0484E+00 -0.2870E+00
* 165 *  0.1900E+00  0.0348E+00 -0.2870E+00

ELEMENTS
INTEG6
*   1*    7    5   13    6   16   14
*   2*   20   22   26   21   27   29
*   3*  141  143  149  142  150  152
*   4*  158  156  162  157  165  163

INTEG8
*   5*    5    3   13   11    4   16   15   12
```

```
*  6*    3    1   11    9    2   15    8   10
*  7*    1   18    9   24   17    8   28   23
*  8*   18   20   24   26   19   28   29   25
*  9*  139  141  147  149  140  151  152  148
* 10*  137  139  145  147  138  144  151  146
* 11*  154  137  160  145  153  164  144  159
* 12*  156  154  162  160  155  165  164  161
* 13*    7   13   43   41   14   33   32   42
* 14*   43   41   63   61   42   53   52   62
* 15*   63   61   83   81   62   73   72   82
* 16*   83   81  103  101   82   93   92  102
* 17*  103  101  123  121  102  113  112  122
* 18*  123  121  143  149  122  133  132  150
* 19*   13   11   41   39   12   32   31   40
* 20*   41   39   61   59   40   52   51   60
* 21*   61   59   81   79   60   72   71   80
* 22*   81   79  101   99   80   92   91  100
* 23*  101   99  121  119  100  112  111  120
* 24*  121  119  149  147  120  132  131  148
* 25*   11    9   39   37   10   31   30   38
* 26*   39   37   59   57   38   51   50   58
* 27*   59   57   79   77   58   71   70   78
* 28*   79   77   99   97   78   91   90   98
* 29*   99   97  119  117   98  111  110  118
* 30*  119  117  147  145  118  131  130  146
* 31*    9   24   37   48   23   30   36   49
* 32*   37   48   57   68   49   50   56   69
* 33*   57   68   77   88   69   70   76   89
* 34*   77   88   97  108   89   90   96  109
* 35*   97  108  117  128  109  110  116  129
* 36*  117  128  145  160  129  130  136  159
* 37*   24   26   48   46   25   36   35   47
* 38*   48   46   68   66   47   56   55   67
* 39*   68   66   88   86   67   76   75   87
* 40*   88   86  108  106   87   96   95  107
* 41*  108  106  128  126  107  116  115  127
* 42*  128  126  160  162  127  136  135  161
* 43*   26   22   46   44   27   35   34   45
* 44*   46   44   66   64   45   55   54   65
* 45*   66   64   86   84   65   75   74   85
* 46*   86   84  106  104   85   95   94  105
* 47*  106  104  126  124  105  115  114  125
* 48*  126  124  162  158  125  135  134  163


EXTERNALPOINT = 100. 100. 100.
MATERIAL= WATER = 0.222E+10  0.1E+04
FREQUENCY = 3000
INTEGRATION = 2,2,0,0
TIMEDEP = -IWT
ALGORITHM MEMORY
SOLVING HELMHOLTZ
IMPEDANCE RADIATION
 REFERENCE / VELOCITY AVERAGE
PROBLEM = 3D
RESULTS = ANGULAR

SYMMETRY
 DECOMPOSITION
  GEOMETRY SYMMETRICAL
   PLANE = XOY
   PLANE = XOZ
   PLANE = YOZ
```

```
  LOAD SYMMETRICAL
   PLANE = XOZ

FARFIELD
 PATTERN BISTATIC
  THETA = 90.                         * PLANE YOZ *
  SPAN / PHI = 0. 180.
  STEP = 5.
 PATTERN BISTATIC                     * PLANE XOZ *
  PHI = 0.
  SPAN / THETA = 0. 180.
  STEP = 5.
 PATTERN MONOSTATIC                   * PLANE XOZ *
  PHI = 0.
  SPAN / THETA = 0. 90.
  STEP = 5

 PROPAGATION SCATTERING
  PLANEWAVE = 1 / 1E-06  90.  0.       * POSITIVE X DIRECTION *

 PROPAGATION RADIATION
  DISPLACEMENT ANALYTICAL
   0.05 0.0 0.1 1E-06


END
```

V.2.2 Data File for Example Case 2

The content of the `case2.eqi` file is fully given below. Please refer to Chapter III for more details about the commands used in this file.

```
* CYLINDER WITH HEMISPHERICAL ENDCAPS
NODES
*  1 *   2.58800E-01     -1.96600E+00
*  2 *   5.00000E-01     -1.86600E+00
*  3 *   3.82700E-01     -1.92400E+00
*  4 *   7.07100E-01     -1.70700E+00
*  5 *   6.08800E-01     -1.79300E+00
*  6 *   8.66000E-01     -1.50000E+00
*  7 *   7.93300E-01     -1.60900E+00
*  8 *   9.65900E-01     -1.25900E+00
*  9 *   9.23900E-01     -1.38300E+00
* 10 *   1.00000E+00     -1.00000E+00
* 11 *   9.91400E-01     -1.13000E+00
* 12 *   0.00000E+00     -2.00000E+00
* 13 *   1.30500E-01     -1.99100E+00
* 14 *   2.58800E-01      1.96600E+00
* 15 *   5.00000E-01      1.86600E+00
* 16 *   3.82700E-01      1.92400E+00
* 17 *   7.07100E-01      1.70700E+00
* 18 *   6.08800E-01      1.79300E+00
* 19 *   8.66000E-01      1.50000E+00
* 20 *   7.93300E-01      1.60900E+00
* 21 *   9.65900E-01      1.25900E+00
* 22 *   9.23900E-01      1.38300E+00
* 23 *   1.00000E+00      1.00000E+00
* 24 *   9.91400E-01      1.13000E+00
* 25 *   0.00000E+00      2.00000E+00
* 26 *   1.30500E-01      1.99100E+00
* 27 *   1.00000E+00     -7.77800E-01
* 28 *   1.00000E+00     -8.88900E-01
* 29 *   1.00000E+00     -5.55500E-01
* 30 *   1.00000E+00     -6.66700E-01
* 31 *   1.00000E+00     -3.33300E-01
* 32 *   1.00000E+00     -4.44400E-01
* 33 *   1.00000E+00     -1.11100E-01
* 34 *   1.00000E+00     -2.22200E-01
* 35 *   1.00000E+00      1.11100E-01
* 36 *   1.00000E+00      0.00000E+00
* 37 *   1.00000E+00      3.33300E-01
* 38 *   1.00000E+00      2.22200E-01
* 39 *   1.00000E+00      5.55600E-01
* 40 *   1.00000E+00      4.44400E-01
* 41 *   1.00000E+00      7.77800E-01
* 42 *   1.00000E+00      6.66700E-01
* 43 *   1.00000E+00      8.88900E-01

ELEMENTS
INTEG3
*  1 * 29 30 27
*  2 *  8  9  6
*  3 * 10 11  8
*  4 * 27 28 10
*  5 * 31 32 29
*  6 * 33 34 31
*  7 * 35 36 33
*  8 * 37 38 35
```

```
*  9 * 39 40 37
* 10 *  1 13 12
* 11 *  2  3  1
* 12 *  4  5  2
* 13 *  6  7  4
* 14 * 41 42 39
* 15 * 23 43 41
* 16 * 19 22 21
* 17 * 17 20 19
* 18 * 15 18 17
* 19 * 14 16 15
* 20 * 25 26 14
* 21 * 21 24 23


PRINTING = 4
EXTERNALPOINT = 100. 100. 100.
MATERIAL = WATER = 0.222E+10  0.1E+04
FREQUENCY = 1400.
INTEGAXI = ELLIPTIC
INTEGRATION = 30,3,6,0
TIMEDEP = -IWT
ALGORITHM MEMORY
SOLVING COUPLING
  PRESSURE = FOURIER
  FOURIER = 20,0
  MAXDEGREE = 1
  ORIGINS
  NUMBER = 3
    0.  0.  1.
    0.  0.  0.
    0.  0. -1.
  CLOSENODES = DESACTIVATION
PROBLEM = AXI
RESULTS = ANGULAR

PROPAGATION SCATTERING
  PLANEWAVE = 1 / 1.E-06 180. 0.

PROPAGATION RADIATION
  DISPLACEMENT INCOMING
    1  0. 0. 0.
    2  0. 0. 0.
    3  0. 0. 0.
    4  0. 0. 0.
    5  0. 0. 0.
    6  0. 0. 0.
    7  0. 0. 0.
    8  0. 0. 0.
    9  0. 0. 0.
   10  0. 0. 0.
   11  0. 0. 0.
   12  0. 0. 0.
   13  0. 0. 0.
   14  0. 0. 0.
   15  0. 0. 0.
   16  0. 0. 0.
   17  0. 0. 0.
   18  0. 0. 0.
   19  0. 0. 0.
   20  0. 0. 0.
   21  0. 0. 0.
   22  0. 0. 0.
```

```
23  0. 0. 0.
24  0. 0. 0.
25  0. 0. 0.
26  0. 0. 0.
27  1.E-09 0. 0.
28  1.E-09 0. 0.
29  1.E-09 0. 0.
30  1.E-09 0. 0.
31  1.E-09 0. 0.
32  1.E-09 0. 0.
33  1.E-09 0. 0.
34  1.E-09 0. 0.
35  1.E-09 0. 0.
36  1.E-09 0. 0.
37  1.E-09 0. 0.
38  1.E-09 0. 0.
39  1.E-09 0. 0.
40  1.E-09 0. 0.
41  1.E-09 0. 0.
42  1.E-09 0. 0.
43  1.E-09 0. 0.
 1  0. 0. 0.
 2  0. 0. 0.
 3  0. 0. 0.
 4  0. 0. 0.
 5  0. 0. 0.
 6  0. 0. 0.
 7  0. 0. 0.
 8  0. 0. 0.
 9  0. 0. 0.
10  0. 0. 0.
11  0. 0. 0.
12  0. 0. 0.
13  0. 0. 0.
14  0. 0. 0.
15  0. 0. 0.
16  0. 0. 0.
17  0. 0. 0.
18  0. 0. 0.
19  0. 0. 0.
20  0. 0. 0.
21  0. 0. 0.
22  0. 0. 0.
23  0. 0. 0.
24  0. 0. 0.
25  0. 0. 0.
26  0. 0. 0.
27  0. 0. 0.
28  0. 0. 0.
29  0. 0. 0.
30  0. 0. 0.
31  0. 0. 0.
32  0. 0. 0.
33  0. 0. 0.
34  0. 0. 0.
35  0. 0. 0.
36  0. 0. 0.
37  0. 0. 0.
38  0. 0. 0.
39  0. 0. 0.
40  0. 0. 0.
41  0. 0. 0.
42  0. 0. 0.
```

```
   43  0. 0. 0.

PROPAGATION RADIATION
  DISPLACEMENT ANALYTIC
    0. 0. 0.3   1.E-06

FARFIELD
  PATTERN BISTATIC
    PHI = 0.
    SPAN / THETA = 0. 180.
    STEP = 5.

NEARFIELD
  PATTERN BISTATIC
    PHI = 0.
    DISTANCE = 5.
    SPAN / THETA = 0. 180.
    STEP = 5.


END
```

V.3 Result Files for the Example Cases

V.3.1 Result File for Example Case 1

To run this example on a PC under Windows95 or 98, you must type

```
eqi602 case1 eqi
```

at the command prompt. The results are contained in a file named `case1.lst`. The full content of this file is given below and briefly commented.

```
1
 On : 12/14/99  at : 09:50:12  Start of job : CASE1

  *********************************************************************
  **** ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN ****
  *********************************************************************
  **                                                                 **
  **                                                                 **
  **  EEEEEEEEEEEEEE        QQQQQQQQQQQQ           IIIIIIIIIIIIIIII  **
  **  EEEEEEEEEEEEEE       QQQQQQQQQQQQQQ          IIIIIIIIIIIIIIII  **
  **  EEE                 QQQQ        QQQQ               IIII        **
  **  EEE                 QQQ          QQQ               IIII        **
  **  EEE                 QQQ          QQQ               IIII        **
  **  EEEEEEEE            QQQ          QQQ               IIII        **
  **  EEEEEEEE            QQQ          QQQ               IIII        **
  **  EEE                 QQQ      QQQ QQQ               IIII        **
  **  EEE                 QQQ       QQQQQQ               IIII        **
  **  EEE                 QQQQ       QQQQQ               IIII        **
  **  EEEEEEEEEEEEEE  **   QQQQQQQQQQQQQQQ     **  IIIIIIIIIIIIIIII  **
  **  EEEEEEEEEEEEEE  **    QQQQQQQQQQQQQQQ    **  IIIIIIIIIIIIIIII  **
  **                                     QQQ                         **
  **                                               ----------------  **
  **                                                Version : 6.02   **
  **                                               ----------------  **
  **                                                                 **
  *********************************************************************
  **** ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN ****
  *********************************************************************

 * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  1


 Reading the data : 09:50:12
 EQIL1 number of nodes IP= 165

 Point of normals direction:   100.000       100.000       100.000


 Propagation medium : WATER
   Medium characteristics :
     Isentropic compressibility coefficient Khi =  2.220000E+09
     Density Rho =   1000.00

 Frequency       =    3000.00       Hz
```

```
 Wavenumber      =      12.6510      /m
 Wavelength      =      0.496655      m
 Sound speed     =      1489.97       m/s

 Number of integration points
 for integral equations   :  2   2
 for null field equations :  0   0

 phase dependence: -IWT

 In-core storage resolution algorithm

 Helmholtz exterior solution

 Radiation impedance computation
 Reference velocity : average normal velocity

 Tridimensional problem

 Results in angular form : magnitude & phase

 Data on symmetries

 Decomposition into elementary problems
 Symmetric geometry with respect to :
     XOY
     XOZ
     YOZ
 Symmetric load with respect to :
     XOZ

 Data on directivity patterns :

 Bistatic pattern # 1
      Constant observation angle theta =      90.00 ø
      Variation of phi =       0.00 ø --->      180.00 ø   step =       5.00 ø
 Bistatic pattern # 2
      Constant observation angle phi =       0.00 ø
      Variation of theta =       0.00 ø --->      180.00 ø   step =       5.00 ø
 Monostatic pattern # 3
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  2

      Constant observation angle phi =       0.00 ø
      Variation of theta =       0.00 ø --->       90.00 ø   step =       5.00 ø

 ==> Load # 1
     Type: rigid scattering
 number of plane waves: 1
 wave # 1   magn =  1.000000E-06  theta =      90.00 ø phi =       0.00 ø

 ==> Load # 2
     Type: point source radiation
 source # 1   X =  5.000000E-02  Y =   0.00000      Z =  0.100000       load =
1.000000E-06

 Verifying the mesh elements: 09:50:12

 Verifying the coherence of the elements normal orientation

 Verifying the external normal
```

```
 Data manipulation : 09:50:12
 Number of active nodes for computation KNDDLT = 165

 Dynamic memory allocation of A         =      37.52  kB
 Dynamic memory =      37.52  kB

 Dynamic memory allocation of B         =       5.16  kB
 Dynamic memory =      42.67  kB

 Dynamic memory allocation of PRSINC    =       2.58  kB
 Dynamic memory =      45.25  kB

 ***********************************************************************

 Elementary problem n : 1    09:50:12

 Basic mesh : number of elements = 12
              number of nodes    = 49

  symmetry plane for loading:
     xOz
     xOy
     yOz

 *---------------------------------------------------------------------*

 Assembling: 09:50:12
 Generating element: INTEG6 , # 1
 Element type: INTEG8 , # 2
 Element type: INTEG8 , # 3
 Element type: INTEG8 , # 4
 Element type: INTEG8 , # 5
 Element type: INTEG8 , # 6
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  3

 Element type: INTEG8 , # 7
 Element type: INTEG8 , # 8
 Element type: INTEG8 , # 9
 Element type: INTEG8 , # 10
 Element type: INTEG8 , # 11
 Element type: INTEG8 , # 12

 Opening on disk the file of monostatic right hand terms : VECMST
   number of records per vector = 1
   number of records = 19
   number of terms per record = 49
   record length = 784  bytes
   disk space used =      14.55  kB

 *---------------------------------------------------------------------*

 Memory solver : 09:50:13

 Dynamic memory allocation of ACOL8     =       6.13  kB
 Dynamic memory =      51.38  kB

 Weigth of A matrix :   27.1014
 Diagonal : lower value   0.465760      upper value   0.788714

   Factorization of A matrix : 09:50:13
 Indicator of conditioning RCOND =  6.669186E-02
```

```
   Determinant of A matrix : 09:50:13

 Abs(Det) =   1.29164     E  -13
   Computation of nodal pressures : 09:50:13
   Computation of monostatic pressures : 09:50:13

  Number of loads solved simultaneously = 8


 Dynamic memory release of ACOL8     =       6.13  kB
 Dynamic memory =      45.25  kB

 Dynamic memory release of A         =      37.52  kB
 Dynamic memory =       7.73  kB

 Processing the data: 09:50:13

 *----------------------------------------------------------------------*

 Pressure in the fluid in the far field: 09:50:13

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      36.06  kB

 Dynamic memory allocation of FDIR      =       1.16  kB
 Dynamic memory =      37.22  kB

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      65.55  kB
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  4


 Dynamic memory allocation of FDIR      =       1.16  kB
 Dynamic memory =      66.70  kB

 Dynamic memory allocation of FMAT      =      14.55  kB
 Dynamic memory =      81.25  kB

 Dynamic memory allocation of FDIR      =       0.30  kB
 Dynamic memory =      81.55  kB
   Assembling: 09:50:13
   Summing: 09:50:14

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      53.22  kB

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      24.89  kB

 Dynamic memory release of FMAT      =      14.55  kB
 Dynamic memory =      10.34  kB

 Destroying the file of monostatic right-hand side vector VECMST

 Data manipulation : 09:50:14

 Dynamic memory allocation of A         =      37.52  kB
 Dynamic memory =      47.86  kB

 **********************************************************************
```

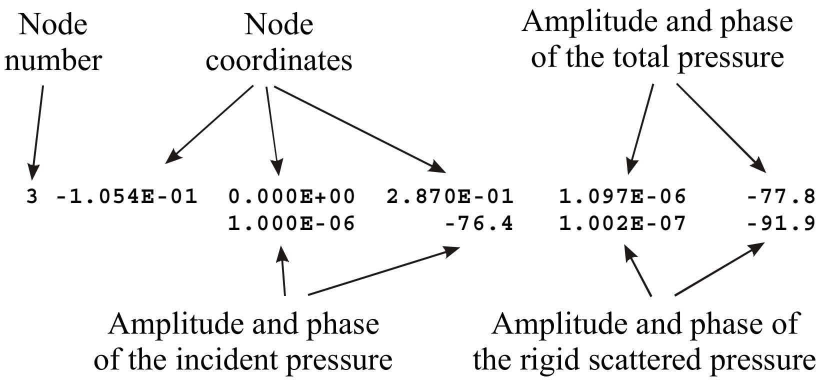
```
 Elementary problem n : 2    09:50:14

  symmetry plane for loading:
     xOz
     xOy

  load's antisymmetry plane :
     yOz

 *----------------------------------------------------------------------*

 Assembling: 09:50:14
 Generating element: INTEG6 , # 1
 Element type: INTEG8 , # 2
 Element type: INTEG8 , # 3
 Element type: INTEG8 , # 4
 Element type: INTEG8 , # 5
 Element type: INTEG8 , # 6
 Element type: INTEG8 , # 7
 Element type: INTEG8 , # 8
 Element type: INTEG8 , # 9
 Element type: INTEG8 , # 10
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  5

 Element type: INTEG8 , # 11
 Element type: INTEG8 , # 12

 Opening on disk the file of monostatic right hand terms : VECMST
   number of records per vector = 1
   number of records = 19
   number of terms per record = 49
   record length = 784  bytes
   disk space used =      14.55  kB

 *----------------------------------------------------------------------*

 Memory solver : 09:50:14

 Dynamic memory allocation of ACOL8     =       6.13  kB
 Dynamic memory =      53.98  kB

 Weigth of A matrix :   27.1373
 Diagonal : lower value   0.458033      upper value   0.790796

   Factorization of A matrix : 09:50:14
 Indicator of conditioning RCOND =  9.620586E-02
   Determinant of A matrix : 09:50:14

 Abs(Det) =   1.02823     E  -13
   Computation of nodal pressures : 09:50:14
   Computation of monostatic pressures : 09:50:14

  Number of loads solved simultaneously = 8


 Dynamic memory release of ACOL8     =       6.13  kB
 Dynamic memory =      47.86  kB

 Dynamic memory release of A         =      37.52  kB
 Dynamic memory =      10.34  kB
```

```
 Processing the data: 09:50:14

 *---------------------------------------------------------------------*

 Pressure in the fluid in the far field: 09:50:14

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      38.67  kB

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      67.00  kB

 Dynamic memory allocation of FMAT      =      14.55  kB
 Dynamic memory =      81.55  kB
   Assembling: 09:50:14
   Summing: 09:50:15

 Dynamic memory release of FMAT      =      28.33  kB
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  6

 Dynamic memory =      53.22  kB

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      24.89  kB

 Dynamic memory release of FMAT      =      14.55  kB
 Dynamic memory =      10.34  kB

 Destroying the file of monostatic right-hand side vector VECMST

 Data manipulation : 09:50:15

 Dynamic memory allocation of A         =      37.52  kB
 Dynamic memory =      47.86  kB

 *********************************************************************

 Elementary problem n : 3    09:50:15

  symmetry plane for loading:
     xOz
     yOz

  load's antisymmetry plane :
     xOy

 *---------------------------------------------------------------------*

 Assembling: 09:50:15
 Generating element: INTEG6 , # 1
 Element type: INTEG8 , # 2
 Element type: INTEG8 , # 3
 Element type: INTEG8 , # 4
 Element type: INTEG8 , # 5
 Element type: INTEG8 , # 6
 Element type: INTEG8 , # 7
 Element type: INTEG8 , # 8
 Element type: INTEG8 , # 9
 Element type: INTEG8 , # 10
 Element type: INTEG8 , # 11
 Element type: INTEG8 , # 12
```

```
 Opening on disk the file of monostatic right hand terms : VECMST
   number of records per vector = 1
   number of records = 19
   number of terms per record = 49
   record length = 784  bytes
   disk space used =      14.55  kB
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  7


 *----------------------------------------------------------------------*

 Memory solver : 09:50:16

 Dynamic memory allocation of ACOL8     =       6.13  kB
 Dynamic memory =      53.98  kB

 Weigth of A matrix :   26.7480
 Diagonal : lower value   0.462115      upper value   0.781376

   Factorization of A matrix : 09:50:16
 Indicator of conditioning RCOND =  7.991517E-02
   Determinant of A matrix : 09:50:16

 Abs(Det) =   8.27495     E  -14
   Computation of nodal pressures : 09:50:16
   Computation of monostatic pressures : 09:50:16

  Number of loads solved simultaneously = 8


 Dynamic memory release of ACOL8     =       6.13  kB
 Dynamic memory =      47.86  kB

 Dynamic memory release of A         =      37.52  kB
 Dynamic memory =      10.34  kB

 Processing the data: 09:50:16

 *----------------------------------------------------------------------*

 Pressure in the fluid in the far field: 09:50:16

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      38.67  kB

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      67.00  kB

 Dynamic memory allocation of FMAT      =      14.55  kB
 Dynamic memory =      81.55  kB
   Assembling: 09:50:16
   Summing: 09:50:17

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      53.22  kB

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      24.89  kB

 Dynamic memory release of FMAT      =      14.55  kB
```

```
 Dynamic memory =      10.34  kB

 Destroying the file of monostatic right-hand side vector VECMST

 Data manipulation : 09:50:17
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  8


 Dynamic memory allocation of A         =      37.52  kB
 Dynamic memory =      47.86  kB

 ***********************************************************************

 Elementary problem n : 4    09:50:17

  symmetry plane for loading:
     xOz

  load's antisymmetry plane :
     xOy
     yOz

 *---------------------------------------------------------------------*

 Assembling: 09:50:17
 Generating element: INTEG6 , # 1
 Element type: INTEG8 , # 2
 Element type: INTEG8 , # 3
 Element type: INTEG8 , # 4
 Element type: INTEG8 , # 5
 Element type: INTEG8 , # 6
 Element type: INTEG8 , # 7
 Element type: INTEG8 , # 8
 Element type: INTEG8 , # 9
 Element type: INTEG8 , # 10
 Element type: INTEG8 , # 11
 Element type: INTEG8 , # 12

 Opening on disk the file of monostatic right hand terms : VECMST
   number of records per vector = 1
   number of records = 19
   number of terms per record = 49
   record length = 784  bytes
   disk space used =      14.55  kB

 *---------------------------------------------------------------------*

 Memory solver : 09:50:18

 Dynamic memory allocation of ACOL8     =       6.13  kB
 Dynamic memory =      53.98  kB

 Weigth of A matrix :   26.9087
 Diagonal : lower value   0.458033      upper value   0.785832

   Factorization of A matrix : 09:50:18
 Indicator of conditioning RCOND =  0.105648
   Determinant of A matrix : 09:50:18

 Abs(Det) =   6.41130     E  -14
1
```

```
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page:  9

   Computation of nodal pressures : 09:50:18
   Computation of monostatic pressures : 09:50:18

  Number of loads solved simultaneously = 8


 Dynamic memory release of ACOL8     =       6.13  kB
 Dynamic memory =      47.86  kB

 Dynamic memory release of A         =      37.52  kB
 Dynamic memory =      10.34  kB

 Processing the data: 09:50:18

 *----------------------------------------------------------------------*

 Printing the results: 09:50:18

 =====> Load # 1

 Rigid scattering:
 Node        X          Y          Z      Magn(Pt)  Phs(Pt)
                   Ampl(Pi)    Phs(Pi)    Magn(Prs) Phs(Prs)
     1  0.000E+00  0.000E+00  2.870E-01   1.064E-06      0.1
                   1.000E-06        0.0   6.396E-08      1.3
     2 -4.840E-02  0.000E+00  2.870E-01   1.065E-06    -36.1
                   1.000E-06      -35.1   6.750E-08    -50.8
     3 -1.054E-01  0.000E+00  2.870E-01   1.097E-06    -77.8
                   1.000E-06      -76.4   1.002E-07    -91.9
```



```
3 -1.054E-01  0.000E+00  2.870E-01   1.097E-06    -77.8
              1.000E-06        -76.4  1.002E-07    -91.9
```

Figure V.I: Reading rigid scattering results

```
     4 -1.441E-01  0.000E+00  2.870E-01   1.112E-06   -105.0
                   1.000E-06     -104.5   1.122E-07   -109.5
     5 -1.900E-01  0.000E+00  2.870E-01   1.108E-06   -138.4
                   1.000E-06     -137.7   1.092E-07   -144.5
     6 -2.161E-01  0.000E+00  2.870E-01   1.114E-06   -158.2
                   1.000E-06     -156.6   1.173E-07   -172.0
     7 -2.420E-01  0.000E+00  2.870E-01   1.185E-06    177.5
                   1.000E-06     -175.4   2.292E-07    144.9
     8  0.000E+00  5.280E-02  2.870E-01   1.058E-06      0.0
                   1.000E-06        0.0   5.761E-08     -0.4
     9  0.000E+00  1.050E-01  2.870E-01   1.051E-06     -0.7
                   1.000E-06        0.0   5.243E-08    -15.1
```

```
    10 -4.840E-02  1.043E-01  2.870E-01   1.073E-06    -38.9
                   1.000E-06      -35.1   1.008E-07    -80.7
    11 -1.054E-01  9.460E-02  2.870E-01   1.135E-06    -81.0
                   1.000E-06      -76.4   1.599E-07   -111.4
    12 -1.441E-01  8.630E-02  2.870E-01   1.155E-06   -108.3
                   1.000E-06     -104.5   1.712E-07   -131.4
    13 -1.900E-01  7.047E-02  2.870E-01   1.155E-06   -141.3
                   1.000E-06     -137.7   1.694E-07   -163.2
    14 -2.273E-01  4.280E-02  2.870E-01   1.166E-06   -170.6
                   1.000E-06     -164.8   1.990E-07    158.9
    15 -1.054E-01  4.840E-02  2.870E-01   1.102E-06    -78.5
                   1.000E-06      -76.4   1.089E-07    -98.4
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page: 10

    16 -1.900E-01  3.480E-02  2.870E-01   1.117E-06   -138.7
                   1.000E-06     -137.7   1.178E-07   -146.8
    17  4.840E-02  0.000E+00  2.870E-01   1.088E-06     34.8
                   1.000E-06       35.1   8.854E-08     31.6
    18  1.054E-01  0.000E+00  2.870E-01   1.083E-06     73.1
                   1.000E-06       76.4   1.019E-07     39.2
    19  1.441E-01  0.000E+00  2.870E-01   1.024E-06     99.0
                   1.000E-06      104.5   9.896E-08     26.0
    20  1.900E-01  0.000E+00  2.870E-01   8.995E-07    134.6
                   1.000E-06      137.7   1.128E-07    -16.8
    21  2.161E-01  0.000E+00  2.870E-01   8.394E-07    159.2
                   1.000E-06      156.6   1.658E-07    -36.5
    22  2.420E-01  0.000E+00  2.870E-01   8.600E-07   -166.6
                   1.000E-06      175.4   3.215E-07    -60.1
    23  4.840E-02  1.043E-01  2.870E-01   1.072E-06     36.5
                   1.000E-06       35.1   7.626E-08     54.8
    24  1.054E-01  9.460E-02  2.870E-01   1.083E-06     76.3
                   1.000E-06       76.4   8.329E-08     74.6
    25  1.441E-01  8.630E-02  2.870E-01   1.027E-06    102.9
                   1.000E-06      104.5   3.876E-08     58.3
    26  1.900E-01  7.047E-02  2.870E-01   9.091E-07    138.9
                   1.000E-06      137.7   9.312E-08    -54.2
    27  2.273E-01  4.280E-02  2.870E-01   8.471E-07    176.7
                   1.000E-06      164.8   2.445E-07    -60.8
    28  1.054E-01  4.840E-02  2.870E-01   1.081E-06     74.0
                   1.000E-06       76.4   9.205E-08     46.9
    29  1.900E-01  3.480E-02  2.870E-01   9.055E-07    135.2
                   1.000E-06      137.7   1.033E-07    -19.6
    30  0.000E+00  1.050E-01  2.645E-01   1.049E-06     -1.8
                   1.000E-06        0.0   5.857E-08    -34.4
    31 -1.054E-01  9.460E-02  2.645E-01   1.187E-06    -86.0
                   1.000E-06      -76.4   2.616E-07   -125.9
    32 -1.900E-01  7.047E-02  2.645E-01   1.226E-06   -148.1
                   1.000E-06     -137.7   3.026E-07    175.2
    33 -2.420E-01  0.000E+00  2.645E-01   1.306E-06    167.6
                   1.000E-06     -175.4   4.548E-07    127.8
    34  2.420E-01  0.000E+00  2.645E-01   9.300E-07   -149.6
                   1.000E-06      175.4   5.834E-07    -70.5
    35  1.900E-01  7.047E-02  2.645E-01   8.781E-07    147.3
                   1.000E-06      137.7   1.986E-07    -89.8
    36  1.054E-01  9.460E-02  2.645E-01   1.089E-06     79.4
                   1.000E-06       76.4   1.046E-07    109.3
    37  0.000E+00  1.050E-01  2.400E-01   1.055E-06     -2.7
                   1.000E-06        0.0   7.406E-08    -42.9
    38 -4.840E-02  1.043E-01  2.400E-01   1.111E-06    -43.5
                   1.000E-06      -35.1   1.908E-07    -94.0
    39 -1.054E-01  9.460E-02  2.400E-01   1.219E-06    -87.6
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                  1.000E-06      -76.4   3.082E-07   -126.9
    40 -1.441E-01  8.630E-02  2.400E-01   1.257E-06   -114.8
                  1.000E-06     -104.5   3.271E-07   -148.3
    41 -1.900E-01  7.047E-02  2.400E-01   1.270E-06   -149.2
                  1.000E-06     -137.7   3.522E-07    176.4
    42 -2.273E-01  4.280E-02  2.400E-01   1.328E-06    178.5
                  1.000E-06     -164.8   4.690E-07    140.7
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
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    43 -2.420E-01  0.000E+00  2.400E-01   1.389E-06    165.1
                  1.000E-06     -175.4   5.569E-07    128.3
    44  2.420E-01  0.000E+00  2.400E-01   9.611E-07   -142.5
                  1.000E-06      175.4   7.056E-07    -70.6
    45  2.273E-01  4.280E-02  2.400E-01   8.563E-07   -162.4
                  1.000E-06      164.8   5.425E-07    -74.1
    46  1.900E-01  7.047E-02  2.400E-01   8.420E-07    148.4
                  1.000E-06      137.7   2.327E-07    -84.4
    47  1.441E-01  8.630E-02  2.400E-01   1.003E-06    106.5
                  1.000E-06      104.5   3.541E-08   -169.3
    48  1.054E-01  9.460E-02  2.400E-01   1.088E-06     78.9
                  1.000E-06       76.4   9.895E-08    105.1
    49  4.840E-02  1.043E-01  2.400E-01   1.086E-06     37.2
                  1.000E-06       35.1   9.422E-08     60.8
    50  0.000E+00  1.050E-01  1.815E-01   1.094E-06     -4.6
                  1.000E-06        0.0   1.262E-07    -44.1
    51 -1.054E-01  9.460E-02  1.815E-01   1.307E-06    -89.4
                  1.000E-06      -76.4   4.021E-07   -123.6
    52 -1.900E-01  7.047E-02  1.815E-01   1.379E-06   -149.7
                  1.000E-06     -137.7   4.510E-07   -177.0
    53 -2.420E-01  0.000E+00  1.815E-01   1.506E-06    165.7
                  1.000E-06     -175.4   6.465E-07    135.6
    54  2.420E-01  0.000E+00  1.815E-01   9.386E-07   -133.2
                  1.000E-06      175.4   8.418E-07    -65.1
    55  1.900E-01  7.047E-02  1.815E-01   7.541E-07    147.8
                  1.000E-06      137.7   2.896E-07    -69.5
    56  1.054E-01  9.460E-02  1.815E-01   1.108E-06     76.0
                  1.000E-06       76.4   1.081E-07     72.4
    57  0.000E+00  1.050E-01  1.210E-01   1.148E-06     -6.1
                  1.000E-06        0.0   1.866E-07    -40.7
    58 -4.840E-02  1.043E-01  1.210E-01   1.226E-06    -47.4
                  1.000E-06      -35.1   3.284E-07    -88.1
    59 -1.054E-01  9.460E-02  1.210E-01   1.399E-06    -89.8
                  1.000E-06      -76.4   4.847E-07   -118.3
    60 -1.441E-01  8.630E-02  1.210E-01   1.459E-06   -115.7
                  1.000E-06     -104.5   5.167E-07   -138.0
    61 -1.900E-01  7.047E-02  1.210E-01   1.470E-06   -147.7
                  1.000E-06     -137.7   5.152E-07   -167.2
    62 -2.273E-01  4.280E-02  1.210E-01   1.514E-06   -178.8
                  1.000E-06     -164.8   5.958E-07    157.0
    63 -2.420E-01  0.000E+00  1.210E-01   1.567E-06    168.3
                  1.000E-06     -175.4   6.682E-07    143.6
    64  2.420E-01  0.000E+00  1.210E-01   8.645E-07   -126.5
                  1.000E-06      175.4   9.134E-07    -58.1
    65  2.273E-01  4.280E-02  1.210E-01   6.839E-07   -149.3
                  1.000E-06      164.8   7.182E-07    -58.4
    66  1.900E-01  7.047E-02  1.210E-01   6.795E-07    141.7
                  1.000E-06      137.7   3.255E-07    -50.5
    67  1.441E-01  8.630E-02  1.210E-01   9.866E-07     97.7
                  1.000E-06      104.5   1.169E-07      4.5
    68  1.054E-01  9.460E-02  1.210E-01   1.150E-06     72.3
                  1.000E-06       76.4   1.692E-07     47.0
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    69  4.840E-02  1.043E-01  1.210E-01   1.170E-06     33.6
                   1.000E-06       35.1   1.722E-07     25.0
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
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    70  0.000E+00  1.050E-01  6.050E-02   1.194E-06     -6.9
                   1.000E-06        0.0   2.341E-07    -37.8
    71 -1.054E-01  9.460E-02  6.050E-02   1.464E-06    -89.6
                   1.000E-06      -76.4   5.410E-07   -114.6
    72 -1.900E-01  7.047E-02  6.050E-02   1.531E-06   -146.0
                   1.000E-06     -137.7   5.599E-07   -160.8
    73 -2.420E-01  0.000E+00  6.050E-02   1.594E-06    171.3
                   1.000E-06     -175.4   6.619E-07    151.0
    74  2.420E-01  0.000E+00  6.050E-02   7.852E-07   -122.1
                   1.000E-06      175.4   9.441E-07    -52.1
    75  1.900E-01  7.047E-02  6.050E-02   6.314E-07    135.3
                   1.000E-06      137.7   3.701E-07    -38.2
    76  1.054E-01  9.460E-02  6.050E-02   1.190E-06     69.4
                   1.000E-06       76.4   2.314E-07     37.9
    77  0.000E+00  1.050E-01  0.000E+00   1.212E-06     -7.2
                   1.000E-06        0.0   2.531E-07    -36.9
    78 -4.840E-02  1.043E-01  0.000E+00   1.298E-06    -48.0
                   1.000E-06      -35.1   3.934E-07    -82.7
    79 -1.054E-01  9.460E-02  0.000E+00   1.489E-06    -89.4
                   1.000E-06      -76.4   5.622E-07   -113.1
    80 -1.441E-01  8.630E-02  0.000E+00   1.554E-06   -114.4
                   1.000E-06     -104.5   5.953E-07   -131.4
    81 -1.900E-01  7.047E-02  0.000E+00   1.553E-06   -145.0
                   1.000E-06     -137.7   5.756E-07   -157.8
    82 -2.273E-01  4.280E-02  0.000E+00   1.566E-06   -175.3
                   1.000E-06     -164.8   6.103E-07    167.4
    83 -2.420E-01  0.000E+00  0.000E+00   1.603E-06    172.1
                   1.000E-06     -175.4   6.633E-07    153.0
    84  2.420E-01  0.000E+00  0.000E+00   7.613E-07   -120.1
                   1.000E-06      175.4   9.617E-07    -50.2
    85  2.273E-01  4.280E-02  0.000E+00   5.561E-07   -146.1
                   1.000E-06      164.8   7.629E-07    -48.7
    86  1.900E-01  7.047E-02  0.000E+00   6.227E-07    132.1
                   1.000E-06      137.7   3.852E-07    -33.2
    87  1.441E-01  8.630E-02  0.000E+00   1.015E-06     91.6
                   1.000E-06      104.5   2.253E-07     11.8
    88  1.054E-01  9.460E-02  0.000E+00   1.208E-06     68.4
                   1.000E-06       76.4   2.586E-07     35.8
    89  4.840E-02  1.043E-01  0.000E+00   1.236E-06     31.6
                   1.000E-06       35.1   2.457E-07     17.5
    90  0.000E+00  1.050E-01 -6.050E-02   1.194E-06     -6.9
                   1.000E-06        0.0   2.341E-07    -37.8
    91 -1.054E-01  9.460E-02 -6.050E-02   1.464E-06    -89.6
                   1.000E-06      -76.4   5.410E-07   -114.6
    92 -1.900E-01  7.047E-02 -6.050E-02   1.531E-06   -146.0
                   1.000E-06     -137.7   5.599E-07   -160.8
    93 -2.420E-01  0.000E+00 -6.050E-02   1.594E-06    171.3
                   1.000E-06     -175.4   6.619E-07    151.0
    94  2.420E-01  0.000E+00 -6.050E-02   7.852E-07   -122.1
                   1.000E-06      175.4   9.441E-07    -52.1
    95  1.900E-01  7.047E-02 -6.050E-02   6.314E-07    135.3
                   1.000E-06      137.7   3.701E-07    -38.2
    96  1.054E-01  9.460E-02 -6.050E-02   1.190E-06     69.4
                   1.000E-06       76.4   2.314E-07     37.9
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
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    97  0.000E+00  1.050E-01 -1.210E-01   1.148E-06      -6.1
                   1.000E-06         0.0   1.866E-07     -40.7
    98 -4.840E-02  1.043E-01 -1.210E-01   1.226E-06     -47.4
                   1.000E-06       -35.1   3.284E-07     -88.1
    99 -1.054E-01  9.460E-02 -1.210E-01   1.399E-06     -89.8
                   1.000E-06       -76.4   4.847E-07    -118.3
   100 -1.441E-01  8.630E-02 -1.210E-01   1.459E-06    -115.7
                   1.000E-06      -104.5   5.167E-07    -138.0
   101 -1.900E-01  7.047E-02 -1.210E-01   1.470E-06    -147.7
                   1.000E-06      -137.7   5.152E-07    -167.2
   102 -2.273E-01  4.280E-02 -1.210E-01   1.514E-06    -178.8
                   1.000E-06      -164.8   5.958E-07     157.0
   103 -2.420E-01  0.000E+00 -1.210E-01   1.567E-06     168.3
                   1.000E-06      -175.4   6.682E-07     143.6
   104  2.420E-01  0.000E+00 -1.210E-01   8.645E-07    -126.5
                   1.000E-06       175.4   9.134E-07     -58.1
   105  2.273E-01  4.280E-02 -1.210E-01   6.839E-07    -149.3
                   1.000E-06       164.8   7.182E-07     -58.4
   106  1.900E-01  7.047E-02 -1.210E-01   6.795E-07     141.7
                   1.000E-06       137.7   3.255E-07     -50.5
   107  1.441E-01  8.630E-02 -1.210E-01   9.866E-07      97.7
                   1.000E-06       104.5   1.169E-07       4.5
   108  1.054E-01  9.460E-02 -1.210E-01   1.150E-06      72.3
                   1.000E-06        76.4   1.692E-07      47.0
   109  4.840E-02  1.043E-01 -1.210E-01   1.170E-06      33.6
                   1.000E-06        35.1   1.722E-07      25.0
   110  0.000E+00  1.050E-01 -1.815E-01   1.094E-06      -4.6
                   1.000E-06         0.0   1.262E-07     -44.1
   111 -1.054E-01  9.460E-02 -1.815E-01   1.307E-06     -89.4
                   1.000E-06       -76.4   4.021E-07    -123.6
   112 -1.900E-01  7.047E-02 -1.815E-01   1.379E-06    -149.7
                   1.000E-06      -137.7   4.510E-07    -177.0
   113 -2.420E-01  0.000E+00 -1.815E-01   1.506E-06     165.7
                   1.000E-06      -175.4   6.465E-07     135.6
   114  2.420E-01  0.000E+00 -1.815E-01   9.386E-07    -133.2
                   1.000E-06       175.4   8.418E-07     -65.1
   115  1.900E-01  7.047E-02 -1.815E-01   7.541E-07     147.8
                   1.000E-06       137.7   2.896E-07     -69.5
   116  1.054E-01  9.460E-02 -1.815E-01   1.108E-06      76.0
                   1.000E-06        76.4   1.081E-07      72.4
   117  0.000E+00  1.050E-01 -2.400E-01   1.055E-06      -2.7
                   1.000E-06         0.0   7.406E-08     -42.9
   118 -4.840E-02  1.043E-01 -2.400E-01   1.111E-06     -43.5
                   1.000E-06       -35.1   1.908E-07     -94.0
   119 -1.054E-01  9.460E-02 -2.400E-01   1.219E-06     -87.6
                   1.000E-06       -76.4   3.082E-07    -126.9
   120 -1.441E-01  8.630E-02 -2.400E-01   1.257E-06    -114.8
                   1.000E-06      -104.5   3.271E-07    -148.3
   121 -1.900E-01  7.047E-02 -2.400E-01   1.270E-06    -149.2
                   1.000E-06      -137.7   3.522E-07     176.4
   122 -2.273E-01  4.280E-02 -2.400E-01   1.328E-06     178.5
                   1.000E-06      -164.8   4.690E-07     140.7
   123 -2.420E-01  0.000E+00 -2.400E-01   1.389E-06     165.1
                   1.000E-06      -175.4   5.569E-07     128.3
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
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   124  2.420E-01  0.000E+00 -2.400E-01   9.611E-07    -142.5
                   1.000E-06       175.4   7.056E-07     -70.6
   125  2.273E-01  4.280E-02 -2.400E-01   8.563E-07    -162.4
                   1.000E-06       164.8   5.425E-07     -74.1
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   126  1.900E-01  7.047E-02 -2.400E-01   8.420E-07    148.4
                   1.000E-06      137.7   2.327E-07    -84.4
   127  1.441E-01  8.630E-02 -2.400E-01   1.003E-06    106.5
                   1.000E-06      104.5   3.541E-08   -169.3
   128  1.054E-01  9.460E-02 -2.400E-01   1.088E-06     78.9
                   1.000E-06       76.4   9.895E-08    105.1
   129  4.840E-02  1.043E-01 -2.400E-01   1.086E-06     37.2
                   1.000E-06       35.1   9.422E-08     60.8
   130  0.000E+00  1.050E-01 -2.645E-01   1.049E-06     -1.8
                   1.000E-06        0.0   5.857E-08    -34.4
   131 -1.054E-01  9.460E-02 -2.645E-01   1.187E-06    -86.0
                   1.000E-06      -76.4   2.616E-07   -125.9
   132 -1.900E-01  7.047E-02 -2.645E-01   1.226E-06   -148.1
                   1.000E-06     -137.7   3.026E-07    175.2
   133 -2.420E-01  0.000E+00 -2.645E-01   1.306E-06    167.6
                   1.000E-06     -175.4   4.548E-07    127.8
   134  2.420E-01  0.000E+00 -2.645E-01   9.300E-07   -149.6
                   1.000E-06      175.4   5.834E-07    -70.5
   135  1.900E-01  7.047E-02 -2.645E-01   8.781E-07    147.3
                   1.000E-06      137.7   1.986E-07    -89.8
   136  1.054E-01  9.460E-02 -2.645E-01   1.089E-06     79.4
                   1.000E-06       76.4   1.046E-07    109.3
   137  0.000E+00  0.000E+00 -2.870E-01   1.064E-06      0.1
                   1.000E-06        0.0   6.396E-08      1.3
   138 -4.840E-02  0.000E+00 -2.870E-01   1.065E-06    -36.1
                   1.000E-06      -35.1   6.750E-08    -50.8
   139 -1.054E-01  0.000E+00 -2.870E-01   1.097E-06    -77.8
                   1.000E-06      -76.4   1.002E-07    -91.9
   140 -1.441E-01  0.000E+00 -2.870E-01   1.112E-06   -105.0
                   1.000E-06     -104.5   1.122E-07   -109.5
   141 -1.900E-01  0.000E+00 -2.870E-01   1.108E-06   -138.4
                   1.000E-06     -137.7   1.092E-07   -144.5
   142 -2.161E-01  0.000E+00 -2.870E-01   1.114E-06   -158.2
                   1.000E-06     -156.6   1.173E-07   -172.0
   143 -2.420E-01  0.000E+00 -2.870E-01   1.185E-06    177.5
                   1.000E-06     -175.4   2.292E-07    144.9
   144  0.000E+00  5.280E-02 -2.870E-01   1.058E-06      0.0
                   1.000E-06        0.0   5.761E-08     -0.4
   145  0.000E+00  1.050E-01 -2.870E-01   1.051E-06     -0.7
                   1.000E-06        0.0   5.243E-08    -15.1
   146 -4.840E-02  1.043E-01 -2.870E-01   1.073E-06    -38.9
                   1.000E-06      -35.1   1.008E-07    -80.7
   147 -1.054E-01  9.460E-02 -2.870E-01   1.135E-06    -81.0
                   1.000E-06      -76.4   1.599E-07   -111.4
   148 -1.441E-01  8.630E-02 -2.870E-01   1.155E-06   -108.3
                   1.000E-06     -104.5   1.712E-07   -131.4
   149 -1.900E-01  7.047E-02 -2.870E-01   1.155E-06   -141.3
                   1.000E-06     -137.7   1.694E-07   -163.2
   150 -2.273E-01  4.280E-02 -2.870E-01   1.166E-06   -170.6
                   1.000E-06     -164.8   1.990E-07    158.9
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
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   151 -1.054E-01  4.840E-02 -2.870E-01   1.102E-06    -78.5
                   1.000E-06      -76.4   1.089E-07    -98.4
   152 -1.900E-01  3.480E-02 -2.870E-01   1.117E-06   -138.7
                   1.000E-06     -137.7   1.178E-07   -146.8
   153  4.840E-02  0.000E+00 -2.870E-01   1.088E-06     34.8
                   1.000E-06       35.1   8.854E-08     31.6
   154  1.054E-01  0.000E+00 -2.870E-01   1.083E-06     73.1
                   1.000E-06       76.4   1.019E-07     39.2
   155  1.441E-01  0.000E+00 -2.870E-01   1.024E-06     99.0
```

```
                   1.000E-06      104.5   9.896E-08     26.0
   156  1.900E-01  0.000E+00 -2.870E-01   8.995E-07    134.6
                   1.000E-06      137.7   1.128E-07    -16.8
   157  2.161E-01  0.000E+00 -2.870E-01   8.394E-07    159.2
                   1.000E-06      156.6   1.658E-07    -36.5
   158  2.420E-01  0.000E+00 -2.870E-01   8.600E-07   -166.6
                   1.000E-06      175.4   3.215E-07    -60.1
   159  4.840E-02  1.043E-01 -2.870E-01   1.072E-06     36.5
                   1.000E-06       35.1   7.626E-08     54.8
   160  1.054E-01  9.460E-02 -2.870E-01   1.083E-06     76.3
                   1.000E-06       76.4   8.329E-08     74.6
   161  1.441E-01  8.630E-02 -2.870E-01   1.027E-06    102.9
                   1.000E-06      104.5   3.876E-08     58.3
   162  1.900E-01  7.047E-02 -2.870E-01   9.091E-07    138.9
                   1.000E-06      137.7   9.312E-08    -54.2
   163  2.273E-01  4.280E-02 -2.870E-01   8.471E-07    176.7
                   1.000E-06      164.8   2.445E-07    -60.8
   164  1.054E-01  4.840E-02 -2.870E-01   1.081E-06     74.0
                   1.000E-06       76.4   9.205E-08     46.9
   165  1.900E-01  3.480E-02 -2.870E-01   9.055E-07    135.2
                   1.000E-06      137.7   1.033E-07    -19.6

 =====> Load # 2

 Point source test:
 Source # 1  X=  5.000000E-02  Y=   0.00000      Z=  0.100000      Load =  1.000000E-
06

 Max error on the phase =   -1.48661      deg  n = 135
 Max error on the magnitude at node:  68  XMom =      -1.78 %
 max analytical magnitude =   9.398043E-06   n = 69

 Mean Square Error MSE =       1.11 %

 *---------------------------------------------------------------------*

 Impedance or selfdiffraction : 09:50:18

 Area of the meshed structure =   0.412373

 Load #  2
 Average velocity on the surface =  -2.737909E-12     9.398043E-06

 Reference velocity =  -2.737909E-12   -2.737909E-12
  Z =  7.1564E+05  -3.2807E+05

 *---------------------------------------------------------------------*

 Pressure in the fluid in the far field: 09:50:18
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page: 16


 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      38.67  kB

 Dynamic memory allocation of FMAT      =      28.33  kB
 Dynamic memory =      67.00  kB

 Dynamic memory allocation of FMAT      =      14.55  kB
 Dynamic memory =      81.55  kB
   Assembling: 09:50:18
```

```
   Summing: 09:50:19

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      53.22  kB

 Dynamic memory release of FMAT      =      28.33  kB
 Dynamic memory =      24.89  kB

 Dynamic memory release of FMAT      =      14.55  kB
 Dynamic memory =      10.34  kB
   Printing the results: 09:50:19

 =====> Load # 1

 Bistatic scattering pattern # 1
 Theta =      90.00 ø max TS =     -13.79  dB for phi =      65.00 ø Ref 1.E-06 Pa at
1m
    Phi f0        Real        Imag        Magn  Phase      dB
     0 0      7.467E-08  8.383E-08  1.123E-07   48.3  -18.99
     5.0      7.145E-08  8.401E-08  1.103E-07   49.6  -19.15
    10.0      6.187E-08  8.451E-08  1.047E-07   53.8  -19.60
```

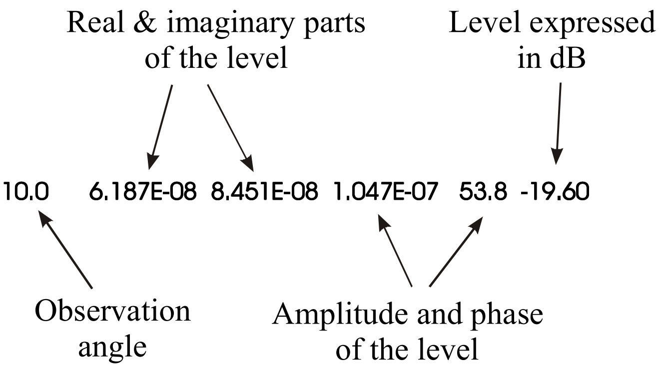

Figure V.2: Reading bistatic scattering pattern results

```
    15.0      4.622E-08  8.531E-08  9.703E-08   61.6  -20.26
    20.0      2.503E-08  8.636E-08  8.991E-08   73.8  -20.92
    25.0     -9.162E-10  8.756E-08  8.757E-08   90.6  -21.15
    30.0     -3.049E-08  8.883E-08  9.392E-08  108.9  -20.54
    35.0     -6.223E-08  9.004E-08  1.095E-07  124.6  -19.22
    40.0     -9.432E-08  9.106E-08  1.311E-07  136.0  -17.65
    45.0     -1.247E-07  9.174E-08  1.548E-07  143.7  -16.21
    50.0     -1.510E-07  9.194E-08  1.768E-07  148.7  -15.05
    55.0     -1.710E-07  9.152E-08  1.940E-07  151.8  -14.25
    60.0     -1.827E-07  9.033E-08  2.038E-07  153.7  -13.82
    65.0     -1.845E-07  8.827E-08  2.045E-07  154.4  -13.79
    70.0     -1.756E-07  8.524E-08  1.952E-07  154.1  -14.19
    75.0     -1.561E-07  8.117E-08  1.759E-07  152.5  -15.09
    80.0     -1.271E-07  7.600E-08  1.481E-07  149.1  -16.59
    85.0     -9.051E-08  6.974E-08  1.143E-07  142.4  -18.84
    90 0     -4.917E-08  6.242E-08  7.946E-08  128.2  -22.00
    95.0     -6.222E-09  5.414E-08  5.449E-08   96.6  -25.27
   100.0      3.512E-08  4.503E-08  5.710E-08   52.0  -24.87
   105.0      7.194E-08  3.529E-08  8.013E-08   26.1  -21.92
   110.0      1.020E-07  2.519E-08  1.051E-07   13.9  -19.57
   115.0      1.239E-07  1.499E-08  1.248E-07    6.9  -18.08
   120.0      1.370E-07  5.002E-09  1.371E-07    2.1  -17.26
   125.0      1.418E-07 -4.508E-09  1.419E-07   -1.8  -16.96
   130.0      1.394E-07 -1.330E-08  1.400E-07   -5.5  -17.08
```

```
  135.0      1.311E-07 -2.120E-08  1.328E-07    -9.2  -17.54
  140.0      1.187E-07 -2.810E-08  1.219E-07   -13.3  -18.28
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page: 17

  145.0      1.039E-07 -3.394E-08  1.093E-07   -18.1  -19.23
  150.0      8.823E-08 -3.876E-08  9.637E-08   -23.7  -20.32
  155.0      7.312E-08 -4.260E-08  8.462E-08   -30.2  -21.45
  160.0      5.956E-08 -4.556E-08  7.498E-08   -37.4  -22.50
  165.0      4.834E-08 -4.772E-08  6.793E-08   -44.6  -23.36
  170.0      4.000E-08 -4.920E-08  6.340E-08   -50.9  -23.96
  175.0      3.487E-08 -5.005E-08  6.099E-08   -55.1  -24.29
  180 0      3.314E-08 -5.032E-08  6.025E-08   -56.6  -24.40

 Bistatic scattering pattern # 2
 Phi =       0.00 ø max TS =     -18.99  dB for theta =      90.00 ø Ref 1.E-06 Pa at
1m
  Theta f0        Real       Imag       Magn   Phase      dB
    0 0      1.638E-08 -1.942E-09  1.649E-08    -6.8  -35.65
    5.0      2.206E-08 -3.027E-09  2.226E-08    -7.8  -33.05
   10.0      2.494E-08 -3.718E-09  2.522E-08    -8.5  -31.97
   15.0      2.414E-08 -3.853E-09  2.444E-08    -9.1  -32.24
   20.0      1.931E-08 -3.219E-09  1.958E-08    -9.5  -34.16
   25.0      1.083E-08 -1.572E-09  1.095E-08    -8.3  -39.21
   30.0     -1.978E-10  1.340E-09  1.355E-09    98.4  -57.36
   35.0     -1.202E-08  5.718E-09  1.331E-08   154.6  -37.52
   40.0     -2.245E-08  1.166E-08  2.530E-08   152.5  -31.94
   45.0     -2.924E-08  1.912E-08  3.494E-08   146.8  -29.13
   50.0     -3.057E-08  2.787E-08  4.136E-08   137.6  -27.67
   55.0     -2.539E-08  3.749E-08  4.528E-08   124.1  -26.88
   60.0     -1.379E-08  4.747E-08  4.943E-08   106.2  -26.12
   65.0      3.011E-09  5.719E-08  5.727E-08    87.0  -24.84
   70.0      2.275E-08  6.605E-08  6.986E-08    71.0  -23.12
   75.0      4.255E-08  7.351E-08  8.494E-08    59.9  -21.42
   80.0      5.940E-08  7.915E-08  9.896E-08    53.1  -20.09
   85.0      7.070E-08  8.265E-08  1.088E-07    49.5  -19.27
   90 0      7.467E-08  8.383E-08  1.123E-07    48.3  -18.99
   95.0      7.070E-08  8.265E-08  1.088E-07    49.5  -19.27
  100.0      5.940E-08  7.915E-08  9.896E-08    53.1  -20.09
  105.0      4.255E-08  7.351E-08  8.494E-08    59.9  -21.42
  110.0      2.275E-08  6.605E-08  6.986E-08    71.0  -23.12
  115.0      3.011E-09  5.719E-08  5.727E-08    87.0  -24.84
  120.0     -1.379E-08  4.747E-08  4.943E-08   106.2  -26.12
  125.0     -2.539E-08  3.749E-08  4.528E-08   124.1  -26.88
  130.0     -3.057E-08  2.787E-08  4.136E-08   137.6  -27.67
  135.0     -2.924E-08  1.912E-08  3.494E-08   146.8  -29.13
  140.0     -2.245E-08  1.166E-08  2.530E-08   152.5  -31.94
  145.0     -1.202E-08  5.718E-09  1.331E-08   154.6  -37.52
  150.0     -1.978E-10  1.340E-09  1.355E-09    98.4  -57.36
  155.0      1.083E-08 -1.572E-09  1.095E-08    -8.3  -39.21
  160.0      1.931E-08 -3.219E-09  1.958E-08    -9.5  -34.16
  165.0      2.414E-08 -3.853E-09  2.444E-08    -9.1  -32.24
  170.0      2.494E-08 -3.718E-09  2.522E-08    -8.5  -31.97
  175.0      2.206E-08 -3.027E-09  2.226E-08    -7.8  -33.05
  180 0      1.638E-08 -1.942E-09  1.649E-08    -6.8  -35.65

 =====> Load # 2

 Radiation pattern # 1

1
```

```
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page: 18

 Max error for phi =     180.00 ø XMom =      -0.85 %
       max analytical value: PMMax =  1.000000E-06  for phi =       0.00 ø

 Dynamic memory release of FDIR      =       1.16  kB
 Dynamic memory =       9.19  kB

 Radiation pattern # 2

 Max error for theta =     180.00 ø XMom =      -1.14 %
       max analytical value: PMMax =  1.000000E-06  for theta =       0.00 ø

 Dynamic memory release of FDIR      =       1.16  kB
 Dynamic memory =       8.03  kB

 Monostatic scattering directivity pattern # 3
 Phi =        0.00 ø max TS =     -13.18  dB for theta =       0.00 ø Ref 1.E-06 Pa at
1m
    Phi f0        Real       Imag       Magn  Phase      dB
     0 0     -2.036E-07 -8.135E-08  2.192E-07 -158.2  -13.18
     5.0     -1.954E-07 -8.096E-08  2.115E-07 -157.5  -13.49
    10.0     -1.712E-07 -7.938E-08  1.887E-07 -155.1  -14.48
    15.0     -1.329E-07 -7.562E-08  1.529E-07 -150.4  -16.31
    20.0     -8.546E-08 -6.871E-08  1.097E-07 -141.2  -19.20
    25.0     -3.828E-08 -5.834E-08  6.978E-08 -123.3  -23.13
    30.0     -2.732E-09 -4.523E-08  4.532E-08  -93.5  -26.87
    35.0      1.250E-08 -3.102E-08  3.344E-08  -68.1  -29.51
    40.0      6.524E-09 -1.767E-08  1.884E-08  -69.7  -34.50
    45.0     -1.210E-08 -6.866E-09  1.391E-08 -150.4  -37.13
    50.0     -2.935E-08  3.174E-10  2.935E-08  179.4  -30.65
    55.0     -3.341E-08  3.316E-09  3.357E-08  174.3  -29.48
    60.0     -2.112E-08  1.842E-09  2.120E-08  175.0  -33.47
    65.0      1.177E-09 -4.121E-09  4.285E-09  -74.1  -47.36
    70.0      2.270E-08 -1.399E-08  2.667E-08  -31.7  -31.48
    75.0      3.533E-08 -2.622E-08  4.399E-08  -36.6  -27.13
    80.0      3.782E-08 -3.823E-08  5.378E-08  -45.3  -25.39
    85.0      3.500E-08 -4.707E-08  5.866E-08  -53.4  -24.63
    90 0      3.314E-08 -5.032E-08  6.025E-08  -56.6  -24.40

 Dynamic memory release of FDIR      =       0.30  kB
 Dynamic memory =       7.73  kB

 Destroying the file of monostatic right-hand side vector VECMST

 Dynamic memory release of PRSINC    =       2.58  kB
 Dynamic memory =       5.16  kB

 Dynamic memory release of B         =       5.16  kB
 Dynamic memory =       0.00  kB

 Destroying the MAIL file of the decomposition in elementary problems

 Destroying the CHPB file of the d,composition in elementary problems

 Destroying the RESP file of the d,composition in elementary problems
1
 Job:CASE1    On:12/14/99  * CYLINDER WITH QUASI-ELLIPTICAL CROSS-SECTION
Page: 19


 Reading data time ........... :        3.30  sec
```

```
         checking the data
         processing the data
         printing the results
Assembling time ............. :        2.80  sec
      of matrix operations
Solving time ................ :        0.39  sec
Impedance calculating time ... :       0.05  sec
Pressure calculating time .... :       3.42  sec
      far-field
Pressure calculating time .... :       0.00  sec
      near-field
Date:12/14/99   09:50:19  End of job:CASE1     CPU time:       9.96  sec
```

V.3.2 Result File for Example Case 2

To run this example on a PC under Windows95 or 98, you must type

```
eqi602 case2 eqi
```

at the command prompt. The results are contained in a file named `case2.lst`. The full content of this file is given below and briefly commented.

```
1
 On : 12/14/99  at : 15:42:55  Start of job : CASE2

  *********************************************************************
  **** ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN ****
  *********************************************************************
  **                                                                 **
  **                                                                 **
  **  EEEEEEEEEEEEEE        QQQQQQQQQQQQ           IIIIIIIIIIIIIIII  **
  **  EEEEEEEEEEEEEE       QQQQQQQQQQQQQQ          IIIIIIIIIIIIIIII  **
  **  EEE                 QQQQ        QQQQ               IIII        **
  **  EEE                 QQQ          QQQ               IIII        **
  **  EEE                 QQQ          QQQ               IIII        **
  **  EEEEEEEE            QQQ          QQQ               IIII        **
  **  EEEEEEEE            QQQ          QQQ               IIII        **
  **  EEE                 QQQ      QQQ QQQ               IIII        **
  **  EEE                 QQQ       QQQQQQ               IIII        **
  **  EEE                 QQQQ       QQQQQ               IIII        **
  **  EEEEEEEEEEEEEE  **   QQQQQQQQQQQQQQQ     **  IIIIIIIIIIIIIIII  **
  **  EEEEEEEEEEEEEE  **    QQQQQQQQQQQQQQQ    **  IIIIIIIIIIIIIIII  **
  **                                     QQQ                         **
  **                                               ----------------  **
  **                                                Version : 6.02   **
  **                                               ----------------  **
  **                                                                 **
  *********************************************************************
  **** ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN-ISEN ****
  *********************************************************************

 * CYLINDER WITH HEMISPHERICAL ENDCAPS
1
 Job:CASE2             On:12/14/99       *   CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
Page:  1


 Reading the data : 15:42:55
 EQIL1 number of nodes IP= 43

 Point of normals direction:   100.000       100.000       100.000


 Propagation medium : WATER
   Medium characteristics :
     Isentropic compressibility coefficient Khi =  2.220000E+09
     Density Rho =   1000.00

 Frequency       =    1400.00       Hz
 Wavenumber      =      5.90380      /m
 Wavelength      =       1.06426      m
 Sound speed     =      1489.97       m/s
```

```
 Integral equations integration in axisymmetrical analysis using elliptical integrals

 Number of integration points
 for integral equations   :  30   3
 for null field equations :  6   0

 phase dependence: -IWT

 In-core storage resolution algorithm

 Coupled solution

 Pressure decomposition using truncated Fourier series

 Number of coefficients in Fourier series decomposition: 21
 according to THETA  P = 0,..., 20
 according to PHI  Q = 0 ,..., 0

 Maximal degree of null field equations =  1

 Multiple origins in null field

 The number of origins is    3

 The  coordinates  of  the  origin        1    are          0.00000                 0.00000
1.00000

 The  coordinates  of  the  origin        2    are          0.00000                 0.00000
0.00000

 The coordinates of the origin      3   are       0.00000            0.00000            -
1.00000

 Desactivation of close nodes method

 Axisymmetrical problem
1
 Job:CASE2              On:12/14/99       *    CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
Page:  2


 Results in angular form : magnitude & phase

 ==> Load # 1
     Type: rigid scattering
 number of plane waves: 1
 wave # 1   magn =  1.000000E-06  theta =     180.00 ø phi =       0.00 ø

 ==> Load # 2
     Type: radiation induced by external displacement

 Dynamic memory allocation of DEPLAX    =       0.67  kB
 Dynamic memory =       0.67  kB

 Dynamic memory allocation of DEPLAY    =       0.67  kB
 Dynamic memory =       1.34  kB

 Dynamic memory allocation of DEPLAZ    =       0.67  kB
 Dynamic memory =       2.02  kB

 ==> Load # 3
     Type: point source radiation
```

```
 source # 1    X =    0.00000       Y =    0.00000       Z =   0.300000        load =
1.000000E-06

 Data on directivity patterns :

 Bistatic pattern # 1
      Constant observation angle phi =       0.00 ø
      Variation of theta =       0.00 ø --->      180.00 ø   step =       5.00 ø

 Data on the FLUID command:

   Pattern #  1
      constant observation angle phi =   0.00000
      distance to observation point =   5.00000
      theta span =   0.00000      --->    180.000        step =   5.00000

 Verifying the mesh elements: 15:42:55

 Data manipulation : 15:42:55
 Number of active nodes for computation KNDDLT = 43

 Dynamic memory allocation of A         =      32.92  kB
 Dynamic memory =      34.94  kB

 Dynamic memory allocation of ZWRK      =       4.03  kB
 Dynamic memory =      38.97  kB

 Dynamic memory allocation of B         =       2.30  kB
 Dynamic memory =      41.27  kB

 Dynamic memory allocation of PRSINC    =       0.77  kB
 Dynamic memory =      42.03  kB
1
 Job:CASE2            On:12/14/99      *   CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
Page:  3


 *----------------------------------------------------------------------*

 Assembling: 15:42:55

 Dynamic memory allocation of CPIAXI    =      28.13  kB
 Dynamic memory =      70.16  kB

 Dynamic memory allocation of CPIAXI    =      28.13  kB
 Dynamic memory =      98.28  kB

 Dynamic memory allocation of CPIAXI    =       0.47  kB
 Dynamic memory =      98.75  kB

 Dynamic memory allocation of CNOAXI    =      28.13  kB
 Dynamic memory =     126.88  kB

 Dynamic memory allocation of CNOAXI    =      28.13  kB
 Dynamic memory =     155.00  kB

 Dynamic memory allocation of CNOAXI    =       0.47  kB
 Dynamic memory =     155.47  kB

 Dynamic memory allocation of XGAXI     =      28.13  kB
 Dynamic memory =     183.59  kB

 Dynamic memory allocation of PGAXI     =      28.13  kB
```

```
 Dynamic memory =     211.72  kB
 Element type: INTEG3 , # 1
 Element type: INTEG3 , # 2
 Element type: INTEG3 , # 3
 Element type: INTEG3 , # 4
 Element type: INTEG3 , # 5
 Element type: INTEG3 , # 6
 Element type: INTEG3 , # 7
 Element type: INTEG3 , # 8
 Element type: INTEG3 , # 9
 Element type: INTEG3 , # 10
 Element type: INTEG3 , # 11
 Element type: INTEG3 , # 12
 Element type: INTEG3 , # 13
 Element type: INTEG3 , # 14
 Element type: INTEG3 , # 15
 Element type: INTEG3 , # 16
 Element type: INTEG3 , # 17
 Element type: INTEG3 , # 18
 Element type: INTEG3 , # 19
 Element type: INTEG3 , # 20
 Element type: INTEG3 , # 21

 Dynamic memory release of PGAXI     =      28.13  kB
 Dynamic memory =     183.59  kB

 Dynamic memory release of XGAXI     =      28.13  kB
 Dynamic memory =     155.47  kB
1
 Job:CASE2             On:12/14/99      *   CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
Page:  4


 Dynamic memory release of CNOAXI    =      28.13  kB
 Dynamic memory =     127.34  kB

 Dynamic memory release of CNOAXI    =      28.13  kB
 Dynamic memory =      99.22  kB

 Dynamic memory release of CNOAXI    =       0.47  kB
 Dynamic memory =      98.75  kB

 Dynamic memory release of CPIAXI    =      28.13  kB
 Dynamic memory =      70.63  kB

 Dynamic memory release of CPIAXI    =      28.13  kB
 Dynamic memory =      42.50  kB

 Dynamic memory release of CPIAXI    =       0.47  kB
 Dynamic memory =      42.03  kB

 Dynamic memory release of ZWRK      =       4.03  kB
 Dynamic memory =      38.00  kB

 *---------------------------------------------------------------------*

 Resolution of overdetermined system : 15:42:58
 System of  49  euations for  43  unkown values

 QR factorization without pivoting : 15:42:58

 Computation of solution by least square method : 15:42:58
```

```
 Load #  1   Average remainder of least square method =   5.227230E-09

 Load #  2   Average remainder of least square method =   5.616118E-03

 Load #  3   Average remainder of least square method =   1.320673E-10


 Dynamic memory release of A         =      32.92  kB
 Dynamic memory =       5.08  kB

 *---------------------------------------------------------------------*

 Printing the results: 15:42:58

 =====> Load # 1

 Rigid scattering:
 Node        X          Y          Z      Magn(Pt)  Phs(Pt)
                   Ampl(Pi)    Phs(Pi)    Magn(Prs) Phs(Prs)
     1  2.588E-01  0.000E+00 -1.966E+00   4.971E-07    147.6
                   1.000E-06      -55.0   1.472E-06    132.5
     2  5.000E-01  0.000E+00 -1.866E+00   5.223E-07     -1.3
                   1.000E-06      -88.8   1.108E-06     63.1
1
 Job:CASE2             On:12/14/99       *   CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
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     3  3.827E-01  0.000E+00 -1.924E+00   3.311E-07     56.0
                   1.000E-06      -69.2   1.221E-06     98.0
```

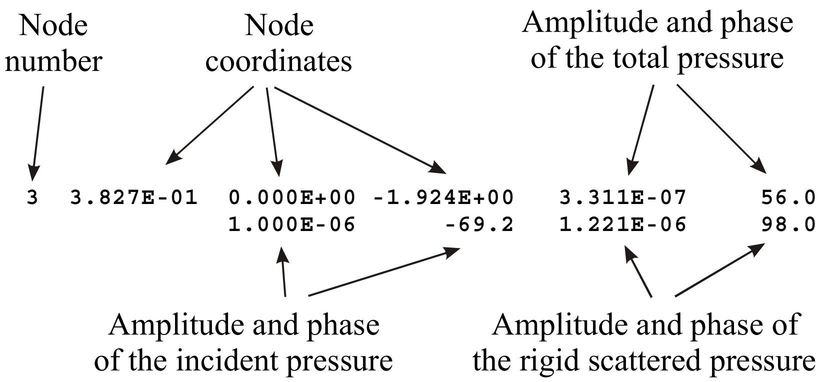

Figure V.3: Reading rigid scattering results

```
     4  7.071E-01  0.000E+00 -1.707E+00   5.958E-07    -89.8
                   1.000E-06     -142.6   7.967E-07      0.8
     5  6.088E-01  0.000E+00 -1.793E+00   5.968E-07    -33.7
                   1.000E-06     -113.5   1.070E-06     33.2
     6  8.660E-01  0.000E+00 -1.500E+00   8.518E-07    171.8
                   1.000E-06      147.4   4.172E-07    -90.1
     7  7.933E-01  0.000E+00 -1.609E+00   6.960E-07   -144.0
                   1.000E-06     -175.7   5.477E-07    -37.6
     8  9.659E-01  0.000E+00 -1.259E+00   8.958E-07     72.6
                   1.000E-06       65.9   1.525E-07   -157.7
     9  9.239E-01  0.000E+00 -1.383E+00   8.754E-07    127.2
                   1.000E-06      107.8   3.388E-07   -131.2
    10  1.000E+00  0.000E+00 -1.000E+00   1.160E-06    -26.9
                   1.000E-06      -21.7   1.871E-07    -55.6
    11  9.914E-01  0.000E+00 -1.130E+00   9.583E-07     18.6
```

```
                   1.000E-06       22.2   7.551E-08   -103.1
    12  0.000E+00  0.000E+00 -2.000E+00   1.136E-06    174.4
                   1.000E-06      -43.5   2.021E-06    156.7
    13  1.305E-01  0.000E+00 -1.991E+00   9.383E-07    168.0
                   1.000E-06      -46.5   1.851E-06    150.2
    14  2.588E-01  0.000E+00  1.966E+00   1.896E-06     43.5
                   1.000E-06       55.0   9.375E-07     31.2
    15  5.000E-01  0.000E+00  1.866E+00   1.829E-06     77.6
                   1.000E-06       88.8   8.697E-07     64.8
    16  3.827E-01  0.000E+00  1.924E+00   1.845E-06     56.5
                   1.000E-06       69.2   8.963E-07     42.4
    17  7.071E-01  0.000E+00  1.707E+00   1.786E-06    131.2
                   1.000E-06      142.6   8.294E-07    117.4
    18  6.088E-01  0.000E+00  1.793E+00   1.778E-06    102.1
                   1.000E-06      113.5   8.216E-07     88.2
    19  8.660E-01  0.000E+00  1.500E+00   1.645E-06   -160.1
                   1.000E-06     -147.4   7.050E-07   -178.3
    20  7.933E-01  0.000E+00  1.609E+00   1.737E-06    163.7
                   1.000E-06      175.7   7.871E-07    148.4
    21  9.659E-01  0.000E+00  1.259E+00   1.417E-06    -76.6
                   1.000E-06      -65.9   4.718E-07    -99.7
    22  9.239E-01  0.000E+00  1.383E+00   1.542E-06   -121.5
                   1.000E-06     -107.8   6.172E-07   -144.0
    23  1.000E+00  0.000E+00  1.000E+00   1.246E-06     21.0
                   1.000E-06       21.7   2.467E-07     17.8
    24  9.914E-01  0.000E+00  1.130E+00   1.240E-06    -28.9
                   1.000E-06      -22.2   2.735E-07    -54.2
    25  0.000E+00  0.000E+00  2.000E+00   1.927E-06     32.7
                   1.000E-06       43.5   9.635E-07     21.4
    26  1.305E-01  0.000E+00  1.991E+00   1.923E-06     34.9
                   1.000E-06       46.5   9.648E-07     22.8
    27  1.000E+00  0.000E+00 -7.778E-01   1.259E-06    -96.2
                   1.000E-06      -96.9   2.590E-07    -93.4
    28  1.000E+00  0.000E+00 -8.889E-01   1.231E-06    -65.1
                   1.000E-06      -59.3   2.564E-07    -88.3
    29  1.000E+00  0.000E+00 -5.555E-01   1.200E-06   -164.4
                   1.000E-06     -172.1   2.481E-07   -131.8
1
 Job:CASE2             On:12/14/99       *    CYLINDER    WITH   HEMISPHERICAL    ENDCAPS
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    30  1.000E+00  0.000E+00 -6.667E-01   1.274E-06   -131.6
                   1.000E-06     -134.5   2.795E-07   -121.3
    31  1.000E+00  0.000E+00 -3.333E-01   9.762E-07    119.2
                   1.000E-06      112.7   1.138E-07   -142.0
    32  1.000E+00  0.000E+00 -4.444E-01   1.137E-06    158.4
                   1.000E-06      150.3   2.033E-07   -157.8
    33  1.000E+00  0.000E+00 -1.111E-01   9.944E-07     30.2
                   1.000E-06       37.6   1.286E-07    -58.6
    34  1.000E+00  0.000E+00 -2.222E-01   9.625E-07     77.3
                   1.000E-06       75.2   5.215E-08   -147.8
    35  1.000E+00  0.000E+00  1.111E-01   1.232E-06    -44.3
                   1.000E-06      -37.6   2.656E-07    -70.3
    36  1.000E+00  0.000E+00  0.000E+00   1.091E-06     -8.1
                   1.000E-06        0.0   1.736E-07    -62.6
    37  1.000E+00  0.000E+00  3.333E-01   1.264E-06   -113.3
                   1.000E-06     -112.7   2.644E-07   -115.4
    38  1.000E+00  0.000E+00  2.222E-01   1.274E-06    -79.1
                   1.000E-06      -75.2   2.847E-07    -93.0
    39  1.000E+00  0.000E+00  5.556E-01   1.249E-06    171.9
                   1.000E-06      172.1   2.493E-07    171.2
    40  1.000E+00  0.000E+00  4.444E-01   1.274E-06   -150.2
                   1.000E-06     -150.3   2.744E-07   -149.9
```
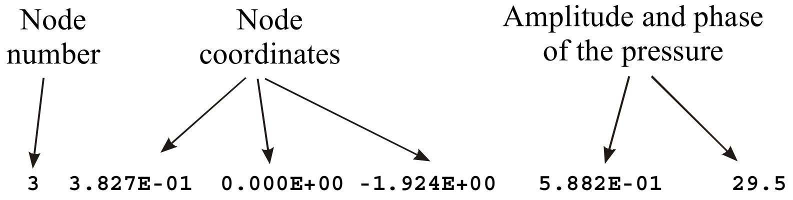

```
    41  1.000E+00  0.000E+00  7.778E-01   1.274E-06     98.0
                   1.000E-06       96.9   2.748E-07    101.8
    42  1.000E+00  0.000E+00  6.667E-01   1.295E-06    135.7
                   1.000E-06      134.5   2.960E-07    140.0
    43  1.000E+00  0.000E+00  8.889E-01   1.262E-06     62.9
                   1.000E-06       59.3   2.706E-07     76.0

 =====> Load # 2


 Node        X          Y          Z        Magn(P)   Phs(P)
     1  2.588E-01  0.000E+00 -1.966E+00   7.886E-01    127.7
     2  5.000E-01  0.000E+00 -1.866E+00   1.067E+00    -13.6
     3  3.827E-01  0.000E+00 -1.924E+00   5.882E-01     29.5
```



```
3  3.827E-01  0.000E+00 -1.924E+00   5.882E-01     29.5
```

Figure V.4: Reading pattern results for the second load

```
     4  7.071E-01  0.000E+00 -1.707E+00   1.099E+00    -97.2
     5  6.088E-01  0.000E+00 -1.793E+00   1.230E+00    -47.8
     6  8.660E-01  0.000E+00 -1.500E+00   1.850E+00    157.2
     7  7.933E-01  0.000E+00 -1.609E+00   1.359E+00   -153.7
     8  9.659E-01  0.000E+00 -1.259E+00   2.607E+00     62.7
     9  9.239E-01  0.000E+00 -1.383E+00   2.198E+00    113.0
    10  1.000E+00  0.000E+00 -1.000E+00   4.567E+00    -58.9
    11  9.914E-01  0.000E+00 -1.130E+00   3.248E+00      6.2
    12  0.000E+00  0.000E+00 -2.000E+00   1.882E+00    157.8
    13  1.305E-01  0.000E+00 -1.991E+00   1.537E+00    152.7
    14  2.588E-01  0.000E+00  1.966E+00   7.886E-01    127.7
    15  5.000E-01  0.000E+00  1.866E+00   1.067E+00    -13.6
    16  3.827E-01  0.000E+00  1.924E+00   5.882E-01     29.5
    17  7.071E-01  0.000E+00  1.707E+00   1.099E+00    -97.2
    18  6.088E-01  0.000E+00  1.793E+00   1.230E+00    -47.8
    19  8.660E-01  0.000E+00  1.500E+00   1.850E+00    157.2
    20  7.933E-01  0.000E+00  1.609E+00   1.359E+00   -153.7
    21  9.659E-01  0.000E+00  1.259E+00   2.607E+00     62.7
1
 Job:CASE2             On:12/14/99       *    CYLINDER    WITH    HEMISPHERICAL    ENDCAPS
Page:   7

    22  9.239E-01  0.000E+00  1.383E+00   2.198E+00    113.0
    23  1.000E+00  0.000E+00  1.000E+00   4.567E+00    -58.9
    24  9.914E-01  0.000E+00  1.130E+00   3.248E+00      6.2
    25  0.000E+00  0.000E+00  2.000E+00   1.882E+00    157.8
    26  1.305E-01  0.000E+00  1.991E+00   1.537E+00    152.7
    27  1.000E+00  0.000E+00 -7.778E-01   1.324E+01   -107.3
    28  1.000E+00  0.000E+00 -8.889E-01   8.836E+00   -108.2
    29  1.000E+00  0.000E+00 -5.555E-01   1.713E+01    -96.6
    30  1.000E+00  0.000E+00 -6.667E-01   1.599E+01   -100.8
    31  1.000E+00  0.000E+00 -3.333E-01   1.451E+01    -92.4
    32  1.000E+00  0.000E+00 -4.444E-01   1.645E+01    -93.3
    33  1.000E+00  0.000E+00 -1.111E-01   9.969E+00    -93.6
    34  1.000E+00  0.000E+00 -2.222E-01   1.193E+01    -92.0
    35  1.000E+00  0.000E+00  1.111E-01   9.969E+00    -93.6
```

```
    36  1.000E+00  0.000E+00  0.000E+00   9.107E+00    -93.4
    37  1.000E+00  0.000E+00  3.333E-01   1.451E+01    -92.4
    38  1.000E+00  0.000E+00  2.222E-01   1.193E+01    -92.0
    39  1.000E+00  0.000E+00  5.556E-01   1.713E+01    -96.6
    40  1.000E+00  0.000E+00  4.444E-01   1.645E+01    -93.3
    41  1.000E+00  0.000E+00  7.778E-01   1.324E+01   -107.3
    42  1.000E+00  0.000E+00  6.667E-01   1.599E+01   -100.8
    43  1.000E+00  0.000E+00  8.889E-01   8.836E+00   -108.2

 =====> Load # 3

 Point source test:
 Source # 1  X=    0.00000       Y=    0.00000       Z=  0.300000       Load =  1.000000E-
06

 Max error on the phase =   0.803091      deg  n = 13
 Max error on the magnitude at node:  34  XMom =      -0.64 %
 max analytical magnitude =   9.994460E-07   n = 37

 Mean Square Error MSE =       0.67 %

 *----------------------------------------------------------------------*

 Pressure in the fluid in the far field: 15:42:58

 Dynamic memory allocation of CPIAXI    =       7.03  kB
 Dynamic memory =      12.11  kB

 Dynamic memory allocation of CPIAXI    =       7.03  kB
 Dynamic memory =      19.14  kB

 Dynamic memory allocation of CPIAXI    =       0.23  kB
 Dynamic memory =      19.38  kB

 Dynamic memory allocation of CNOAXI    =       7.03  kB
 Dynamic memory =      26.41  kB

 Dynamic memory allocation of CNOAXI    =       7.03  kB
 Dynamic memory =      33.44  kB

 Dynamic memory allocation of CNOAXI    =       0.23  kB
1
 Job:CASE2            On:12/14/99       *   CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
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 Dynamic memory =      33.67  kB

 Dynamic memory allocation of XGAXI     =       7.03  kB
 Dynamic memory =      40.70  kB

 Dynamic memory allocation of PGAXI     =       7.03  kB
 Dynamic memory =      47.73  kB

 Dynamic memory allocation of FMAT      =      24.86  kB
 Dynamic memory =      72.59  kB

 Dynamic memory allocation of FDIR      =       1.73  kB
 Dynamic memory =      74.33  kB
   Assembling: 15:42:58
   Summing: 15:42:59

 Dynamic memory release of FMAT      =      24.86  kB
 Dynamic memory =      49.47  kB
```

```
 Dynamic memory release of PGAXI     =       7.03  kB
 Dynamic memory =      42.44  kB

 Dynamic memory release of XGAXI     =       7.03  kB
 Dynamic memory =      35.41  kB

 Dynamic memory release of CNOAXI    =       7.03  kB
 Dynamic memory =      28.38  kB

 Dynamic memory release of CNOAXI    =       7.03  kB
 Dynamic memory =      21.34  kB

 Dynamic memory release of CNOAXI    =       0.23  kB
 Dynamic memory =      21.11  kB

 Dynamic memory release of CPIAXI    =       7.03  kB
 Dynamic memory =      14.08  kB

 Dynamic memory release of CPIAXI    =       7.03  kB
 Dynamic memory =       7.05  kB

 Dynamic memory release of CPIAXI    =       0.23  kB
 Dynamic memory =       6.81  kB
   Printing the results: 15:42:59

 =====> Load # 1

 Bistatic scattering pattern # 1
 Phi =       0.00 ø max TS =       6.96  dB for theta =     180.00 ø Ref 1.E-06 Pa at
1m
  Theta f0        Real        Imag        Magn  Phase      dB
    0 0      1.689E-07  3.961E-07  4.306E-07   66.9   -7.32
    5.0      1.529E-07  4.048E-07  4.327E-07   69.3   -7.28
   10.0      1.048E-07  4.284E-07  4.410E-07   76.3   -7.11
   15.0      2.533E-08  4.578E-07  4.585E-07   86.8   -6.77
   20.0     -8.158E-08  4.756E-07  4.825E-07   99.7   -6.33
   25.0     -2.054E-07  4.552E-07  4.993E-07  114.3   -6.03
1
 Job:CASE2              On:12/14/99       *    CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
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   30.0     -3.256E-07  3.666E-07  4.903E-07  131.6   -6.19
   35.0     -4.109E-07  1.909E-07  4.531E-07  155.1   -6.88
   40.0     -4.228E-07 -5.832E-08  4.268E-07 -172.1   -7.40
   45.0     -3.262E-07 -3.198E-07  4.568E-07 -135.6   -6.80
   50.0     -1.101E-07 -4.936E-07  5.057E-07 -102.6   -5.92
   55.0      1.885E-07 -4.859E-07  5.212E-07  -68.8   -5.66
   60.0      4.713E-07 -2.754E-07  5.458E-07  -30.3   -5.26
   65.0      5.997E-07  4.490E-08  6.014E-07    4.3   -4.42
   70.0      4.672E-07  2.970E-07  5.536E-07   32.4   -5.14
   75.0      9.240E-08  3.224E-07  3.353E-07   74.0   -9.49
   80.0     -3.406E-07  9.502E-08  3.536E-07  164.4   -9.03
   85.0     -5.697E-07 -2.486E-07  6.216E-07 -156.4   -4.13
   90 0     -4.468E-07 -5.001E-07  6.706E-07 -131.8   -3.47
   95.0     -8.396E-08 -5.234E-07  5.301E-07  -99.1   -5.51
  100.0      2.044E-07 -3.390E-07  3.959E-07  -58.9   -8.05
  105.0      1.591E-07 -8.589E-08  1.808E-07  -28.4  -14.85
  110.0     -1.857E-07  9.095E-08  2.068E-07  153.9  -13.69
  115.0     -5.176E-07  1.293E-07  5.335E-07  166.0   -5.46
  120.0     -5.187E-07  5.852E-08  5.220E-07  173.6   -5.65
  125.0     -1.505E-07 -4.918E-08  1.584E-07 -161.9  -16.01
  130.0      3.098E-07 -1.273E-07  3.349E-07  -22.3   -9.50
```

```
  135.0      4.992E-07 -1.326E-07  5.165E-07  -14.9   -5.74
  140.0      2.523E-07 -4.073E-08  2.555E-07   -9.2  -11.85
  145.0     -2.932E-07  1.573E-07  3.327E-07  151.8   -9.56
  150.0     -8.156E-07  4.514E-07  9.322E-07  151.0   -0.61
  155.0     -1.027E-06  8.095E-07  1.307E-06  141.7    2.33
  160.0     -8.233E-07  1.184E-06  1.442E-06  124.8    3.18
  165.0     -3.057E-07  1.523E-06  1.554E-06  101.3    3.83
  170.0      3.072E-07  1.788E-06  1.815E-06   80.3    5.18
  175.0      7.845E-07  1.955E-06  2.106E-06   68.1    6.47
  180 0      9.630E-07  2.011E-06  2.230E-06   64.4    6.96

 =====> Load # 2

 Radiation pattern # 1
 Phi =        0.00 ø max level =      147.60  dB for theta =       90.00 ø Ref 1.E-06 Pa
at 1m
    Phi f0         Real       Imag       Magn  Phase      dB
    0 0      8.379E+00 -1.527E+00  8.517E+00  -10.3  138.61
    5.0      7.976E+00 -1.805E+00  8.177E+00  -12.8  138.25
   10.0      6.692E+00 -2.574E+00  7.170E+00  -21.0  137.11
   15.0      4.423E+00 -3.626E+00  5.719E+00  -39.3  135.15
   20.0      1.280E+00 -4.601E+00  4.776E+00  -74.5  133.58
   25.0     -2.219E+00 -5.036E+00  5.503E+00 -113.8  134.81
   30.0     -5.208E+00 -4.514E+00  6.892E+00 -139.1  136.77
   35.0     -6.823E+00 -2.907E+00  7.417E+00 -156.9  137.40
   40.0     -6.640E+00 -5.871E-01  6.666E+00 -174.9  136.48
   45.0     -4.917E+00  1.557E+00  5.158E+00  162.4  134.25
   50.0     -2.479E+00  2.418E+00  3.463E+00  135.7  130.79
   55.0     -3.161E-01  1.180E+00  1.222E+00  105.0  121.74
   60.0      7.946E-01 -2.225E+00  2.362E+00  -70.3  127.47
   65.0      4.545E-01 -7.048E+00  7.062E+00  -86.3  136.98
   70.0     -1.361E+00 -1.209E+01  1.217E+01  -96.4  141.71
   75.0     -4.304E+00 -1.631E+01  1.687E+01 -104.8  144.54
   80.0     -7.626E+00 -1.916E+01  2.063E+01 -111.7  146.29
1
 Job:CASE2             On:12/14/99      *    CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
Page: 10

   85.0     -1.028E+01 -2.069E+01  2.311E+01 -116.4  147.27
   90 0     -1.129E+01 -2.116E+01  2.398E+01 -118.1  147.60
   95.0     -1.028E+01 -2.069E+01  2.311E+01 -116.4  147.27
  100.0     -7.626E+00 -1.916E+01  2.063E+01 -111.7  146.29
  105.0     -4.304E+00 -1.631E+01  1.687E+01 -104.8  144.54
  110.0     -1.361E+00 -1.209E+01  1.217E+01  -96.4  141.71
  115.0      4.545E-01 -7.048E+00  7.062E+00  -86.3  136.98
  120.0      7.946E-01 -2.225E+00  2.362E+00  -70.3  127.47
  125.0     -3.161E-01  1.181E+00  1.222E+00  105.0  121.74
  130.0     -2.479E+00  2.418E+00  3.463E+00  135.7  130.79
  135.0     -4.917E+00  1.557E+00  5.158E+00  162.4  134.25
  140.0     -6.640E+00 -5.872E-01  6.666E+00 -174.9  136.48
  145.0     -6.823E+00 -2.907E+00  7.417E+00 -156.9  137.40
  150.0     -5.208E+00 -4.514E+00  6.892E+00 -139.1  136.77
  155.0     -2.219E+00 -5.036E+00  5.503E+00 -113.8  134.81
  160.0      1.280E+00 -4.601E+00  4.776E+00  -74.5  133.58
  165.0      4.423E+00 -3.626E+00  5.719E+00  -39.3  135.15
  170.0      6.692E+00 -2.574E+00  7.170E+00  -21.0  137.11
  175.0      7.976E+00 -1.805E+00  8.177E+00  -12.8  138.25
  180 0      8.379E+00 -1.527E+00  8.517E+00  -10.3  138.61

 =====> Load # 3

 Radiation pattern # 1
```

```
 Max error for theta =     115.00 ø XMom =       0.82 %
       max analytical value: PMMax =  1.000000E-06  for theta =       0.00 ø

 Dynamic memory release of FDIR      =       1.73  kB
 Dynamic memory =       5.08  kB

 Dynamic memory release of PRSINC    =       0.77  kB
 Dynamic memory =       4.31  kB

 Dynamic memory release of B         =       2.30  kB
 Dynamic memory =       2.02  kB

 Dynamic memory release of DEPLAZ    =       0.67  kB
 Dynamic memory =       1.34  kB

 Dynamic memory release of DEPLAY    =       0.67  kB
 Dynamic memory =       0.67  kB

 Dynamic memory release of DEPLAX    =       0.67  kB
 Dynamic memory =       0.00  kB
1
 Job:CASE2             On:12/14/99      *   CYLINDER   WITH   HEMISPHERICAL   ENDCAPS
Page: 14


 Reading data time ........... :        0.38  sec
          checking the data
          processing the data
          printing the results
 Assembling time ............. :        2.86  sec
       of matrix operations
 Solving time ................ :        0.05  sec
 Impedance calculating time ... :       0.00  sec
 Pressure calculating time .... :       0.98  sec
       far-field
 Pressure calculating time .... :       1.60  sec
       near-field
 Date:12/14/99   15:43:01  End of job:CASE2     CPU time:       5.87  sec
```

V.4 Examples of EQI-Atila coupling

In this chapter, we briefly show two examples that are solved by using the **EQI-Atila** coupling. The version of **Atila** used is version 5.2.1.

V.4.1 Example 1: sph3d

The initial file, named `sph3d.ati`, is created. It contains a composite spherical shell surrounded with surface impedance elements. A partial listing of this file is given below. This file must be a valid Atila data file, created using **MOSAIQUE**, **PREFLU** & **PREATI**, or other preprocessor. The special requirements on this file, in prevision of using it for an **EQI-Atila** coupled computation, are:

- The fluid domain must not be meshed; it is completely ignored in the **Atila** data file
- The solid-fluid interface must be meshed using the surface impedance elements (**LINE03Z**, **TRIA06Z**, and **QUAD08Z**)

```
* SPHERE
ANALYSIS HARMONIC
EQI
NLOAD
  1
LCPDDC
  8
FREQUENCY = 2050
NEWAXES SPHERICAL
 0.  0.  0.  0.  0.  0.
GEOMETRY
   1
   1  0.0  0.03

MATERIAL
  CAU4G
   0.714E+11  0.344  2780.  0.   0.  0. &
   0.714E+11  0.344  2780.  0.   1.  0.

NODES
*    1 *  0.985  90.0   0.0
...
*  321 *  0.985   0.0   0.0

NEWAXES SPHERICAL
 0.  0.   0.  -90.  180.  0.
NODES
*    1 *  0.985  90.0   0.0
...
*  321 *  0.985   0.0   0.0

ELEMENTS
SHEL08C CAU4G  1
*  * 1   3   33  35  2   22  23  34
...
*  * 275 277 307 309 276 287 288 308
```

```
SHEL06C CAU4G  1
*  * 289 291 321 290 311 310
...
*  * 307 309 321 308 320 319

SHEL08C CAU4G   1
*  * 21 19 354 356 20 343 344 355
...
*  * 596 598 628 630 597 608 609 629

SHEL06C CAU4G  1
*  * 610 612 642 611 632 631
...
*  * 628 630 642 629 641 640

QUAD08Z
*  * 1   3   33  35  2   22  23  34
...
*  * 275 277 307 309 276 287 288 308

TRIA06Z
*  * 289 291 321 290 311 310
...
*  * 307 309 321 308 320 319

QUAD08Z
*  * 21 19 354 356 20 343 344 355
...
*  * 596 598 628 630 597 608 609 629

TRIA06Z
*  * 610 612 642 611 632 631
...
*  * 628 630 642 629 641 640


END
 -321   15    1
 -321   24    2
```

You can use the **Display Mesh** feature of the **Atila 5.2.1** supervisor to see the mesh on screen. The result is shown in Fig. V.5. Because this file contains surface impedance elements (**TRIA06Z** and **QUAD08Z**), the mesh visualization feature also results in the automatic creation of a file, named `sph3d.eqil`, located in the current working directory. A partial listing of this file is given below.

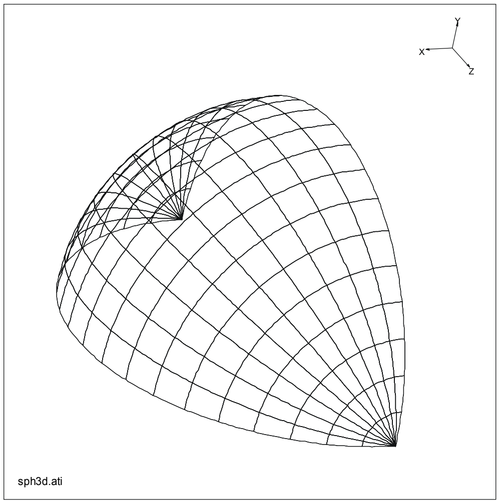


Figure V.5: Geometry of the composite spherical shell displayed by using the Mesh Drawing feature of Atila 5.2.1

The `sph3d.eqil` file only contains the listing of the nodes and elements corresponding to the interface between the solid and fluid regions. This interface was identified by the surface impedance elements in the `sph3d.ati` file.

```
*   1*  0.9850       0.000       0.000
...
* 621*  7.7282E-02   0.000      -0.9820
* 201*    1    2    3    4    5    6    7    8
...
* 290*  272  275  304  307  277  305  308  309
* 291*  278  279  310  282  311  312
...
* 300*  304  307  310  309  321  320
* 301*   49   44  322  323   51  324  325  326
...
* 390*  572  575  604  607  577  605  608  609
* 391*  578  579  610  582  611  612
...
* 400*  604  607  610  609  621  620
```

Your next task is to edit and modify this file in order to generate the desired `sph3d.eqi` file, which will be used by **EQI**. A partial listing of this file is given below.

```
NODES
*   1*  0.1000E+01  0.0000E+00  0.0000E+00
...
* 621*  0.7846E-01  0.0000E+00 -0.9969E+00

ELEMENTS
INTEG8
```

```
* 201*    1    2    3    4    5    6    7    8
...
* 290*  272  275  304  307  277  305  308  309

INTEG6
* 291*  278  279  310  282  311  312
...
* 300*  304  307  310  309  321  320

INTEG8
* 301*   49   44  322  323   51  324  325  326
...
* 390*  572  575  604  607  577  605  608  609

INTEG6
* 391*  578  579  610  582  611  612
...
* 400*  604  607  610  609  621  620


EXTERNALPOINT = 100. 100. 100.
MATERIAL = WATER = 2.2200E+09 0.1E+04
SOLVING HELMHOLTZ
FREQUENCY = 2050
INTEGRATION = 2,2,0,0
TIMEDEP = +IWT
ALGORITHM MEMORY
GENERATION POST-ATI
PROBLEM = 3D

SYMMETRY
 GEOMETRY SYMMETRIC
 PLANE = YOZ
 PLANE = XOZ
 LOAD SYMMETRIC
 PLANE = YOZ
 PLANE = XOZ

 FARFIELD
   PATTERN BISTATIC
   PHI = 0.                         * PLANE XOZ *
   SPAN / THETA = 0. 180.
   STEP = 1.

 PROPAGATION SCATTERING
   PLANEWAVE = 1 / 1.E-06  0. 0.

 PROPAGATION RADIATION
   DISPLACEMENT ANALYTIC
     0.25  0.25  0.25  1.E-06
     0.25 -0.25  0.25  1.E-06
    -0.25  0.25  0.25  1.E-06
    -0.25 -0.25  0.25  1.E-06


END
```

Once this file is created, you can solve the problem. This is simply done by starting the **Solver** from the **Atila** supervisor. **Atila 5.2.1** takes care of running all the steps of the computation, including the **EQI** modules.

Once the computation is completed, the **Atila** results can be found in the `sph3d.lst` file, and the

EQI results in the `sph3d.lps2050` file, as well as in `sph3d.lstef`.

Also, in the `sph3d` subdirectory, a file named `sph3d.pat2050` is created with the pattern information. To use this file to display the directivity patterns using **Atila** postprocessors, you must edit this file. A partial listing of the `sph3d.pat2050` is given below.

```
scattered pressure pattern
 1 'Bi  Phi =       0.00 ø max TS =      14.16  dB for theta =       0.00 ø'  2
      1  2.0500E+03
      181  'DECB'
    0.0      14.16
...
  180.0      -5.39
   radiated pressure pattern
 1 'Bi  Phi =       0.00 ø max TS =      10.63  dB for theta =       0.00 ø'  2
      1  2.0500E+03
      181  'DECB'
    0.0      10.63
...
  180.0      -1.01
   rigid scattered pressure pattern
 1 'Bi  Phi =       0.00 ø max TS =      10.80  dB for theta =       0.00 ø'  2
      1  2.0500E+03
      181  'DECB'
    0.0      10.80
...
  180.0      -5.80
```

To be able to view the directivity pattern using **Atila**, you must modify this file as indicated below, and save it as `sph3d.pat`.

```
1 'Bi  Phi =       0.00 ø max TS =      14.16  dB for theta =       0.00 ø'  2
      3  2.0500E+03
      181  'DECB'
    0.0      14.16
...
  180.0      -5.39
      181  'DECB'
    0.0      10.63
...
  180.0      -1.01
      181  'DECB'
    0.0      10.80
...
  180.0      -5.80
```

The modifications are:

- Remove the initial line of text
- Modify line two to indicate that there are 3 patterns in the file
- Remove the extra text lines in between the patterns

Finally, you can display the directivity patterns by selecting the **Directivities** feature from the **Atila** supervisor. You can also display the deformation of the composite shell by using the **Display Mesh** feature, and selecting the **Mesh Deformation** option for **2,050 Hz**. The results are shown in Fig. V.6 and Fig. V.7.

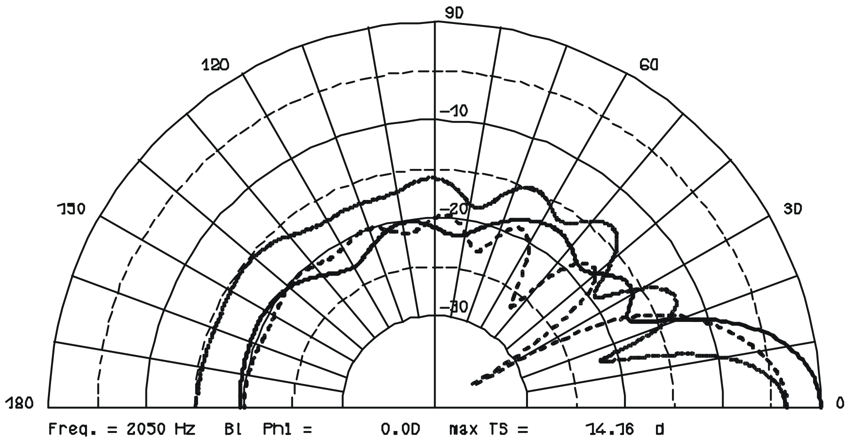


Figure V.6: Directivity patterns for example sph3d

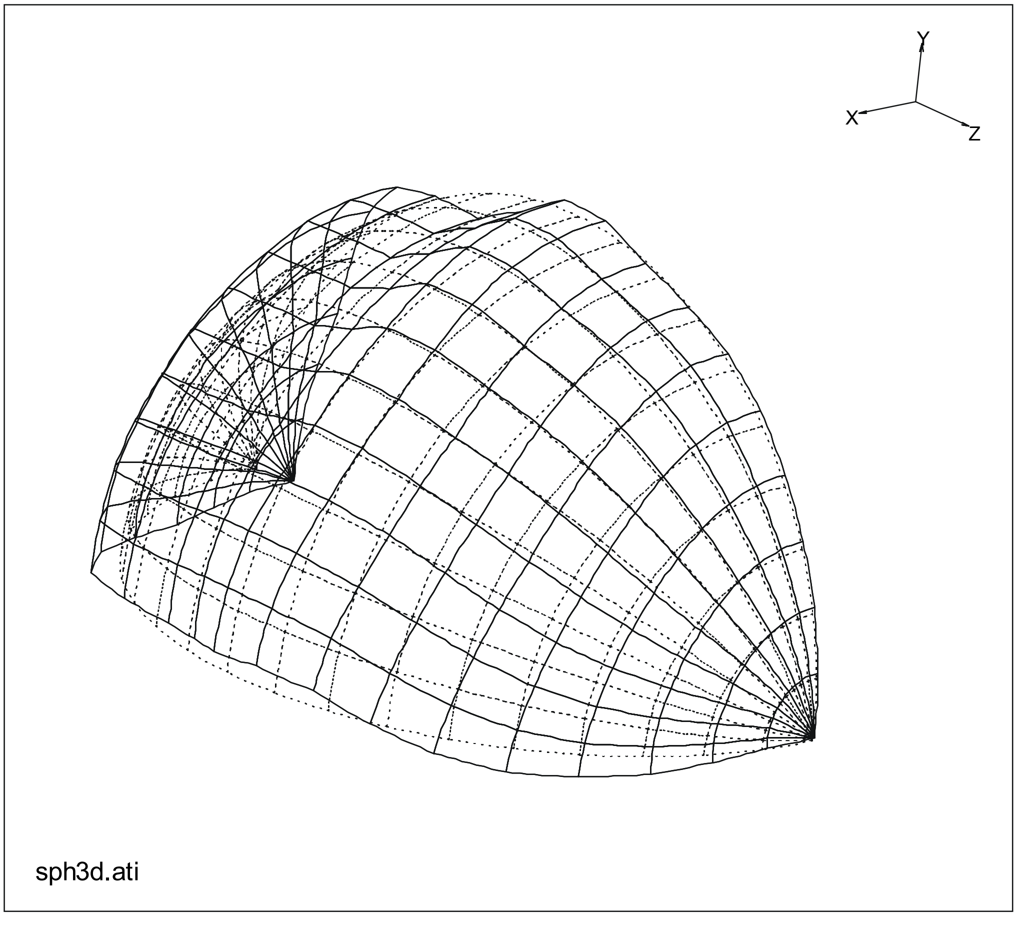


Figure V.7: Real part of the shell deformation at 2,050 Hz

V.4.2 Example 2: sph2d

The second example is a 2D axially symmetric representation of example 1. The file is named `sph2d.ati`, and a partial listing is shown below. Also, the mesh displayed using **Atila**'s **Display Mesh** feature is shown in Fig. V.8.

```
* SCATTERING BY A SPHERICAL SHELL (AXIAL SYMMETRY)
ANALYSIS HARMONIC
CLASS AXISYMMETRICAL
EQI
NLOAD
  1
LCPDDC
  9
FREQUENCY = 2050
NEWAXES SPHERICAL
 0.  0.  0.  0.  -90.  -90.
GEOMETRY
   1
   0.03   0.985

NODES
*   41 *  0.985   180.0    0.
...
*    1 *  0.985     0.0    0.

ELEMENTS
SHEL03E AU4G  1
*  * 1   2   3
...
*  * 39  40  41

LINE03Z
*  * 1   3   2
...
*  * 39  41  40


END
    0  345    0
   -1   26    4
```

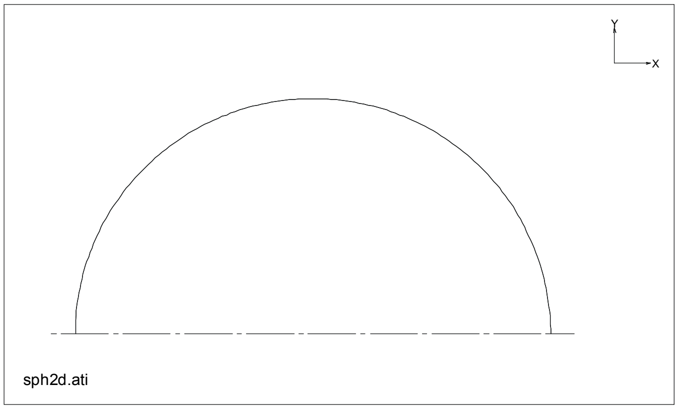


Figure V.8: Display of the mesh in sph2d.ati

The `sph2d.eqi1` is given below. This file is automatically created by **Atila** when you display the mesh.

```
*   1*   0.000     -0.9850
...
*  41*  7.7282E-02  0.9820
*  21*    2    3    1
...
*  40*   40   41   38
```

From this file, you can create a `sph2d.eqi` file:

```
NODES
*   1*   0.000     -0.9850
...
*  41*  7.7282E-02  0.9820

ELEMENTS
INTEG3
*  21*    2    3    1
...
*  40*   40   41   38


EXTERNALPOINT = 100. 100. 100.
MATERIAL = WATER = 2.2200E+09 0.1E+04 /
ALGORITHM MEMORY
SOLVING HELMHOLTZ
FREQUENCY = 2050
INTEGRATION = 23,3,0,0
TIMEDEP = +IWT
GENERATION POST-ATI
PROBLEM = AXI
RESULTS = CARTESIAN

 FARFIELD
   PATTERN BISTATIC
   PHI = 0.                            * PLANE XOZ *
   SPAN / THETA = 0. 180.
   STEP = 1.

 PROPAGATION SCATTERING
   PLANEWAVE = 1 / 1.E-06  0. 0.

 PROPAGATION RADIATION
   DISPLACEMENT ANALYTIC
     0.00  0.00  0.5   1.E-06
     0.00  0.00 -0.5   1.E-06

 PROPAGATION RADIATION
   DISPLACEMENT ANALYTIC
     0.00  0.00  0.5   1.E-06
     0.00  0.00 -0.5  -1.E-06


END
```

After solving the coupled problem, you obtain, in the `sph2d` subdirectory, a file named `shp2d.pat2050`. This file is, in part, shown below.

```
scattered pressure pattern
 1 'Bi  Phi =       0.00 ø max TS =      14.00  dB for theta =       0.00 ø'  2
      1  2.0500E+03
      181  'DECB'
    0.0      14.00
...
  180.0     -16.56
   radiated pressure pattern
 1 'Bi  Phi =       0.00 ø max TS =      11.17  dB for theta =       0.00 ø'  2
      1  2.0500E+03
      181  'DECB'
    0.0      11.17
...
  180.0      -3.85
   rigid scattered pressure pattern
 1 'Bi  Phi =       0.00 ø max TS =      10.53  dB for theta =       0.00 ø'  2
      1  2.0500E+03
      181  'DECB'
    0.0      10.53

  180.0      -5.72
```

You must modify this file, and save it as sph2d.pat, in order to view the directivity patterns from the Atila supervisor. The modifications are shown below.

```
1 'Bi  Phi =       0.00 ø max TS =      14.00  dB for theta =       0.00 ø'  2
      3  2.0500E+03
      181  'DECB'
    0.0      14.00
...
  180.0     -16.56
      181  'DECB'
    0.0      11.17
...
  180.0      -3.85
      181  'DECB'
    0.0      10.53

  181.0      -5.72
```

The directivity patterns, as well as the structure deformation, are shown in Figure V.9 and V.10.

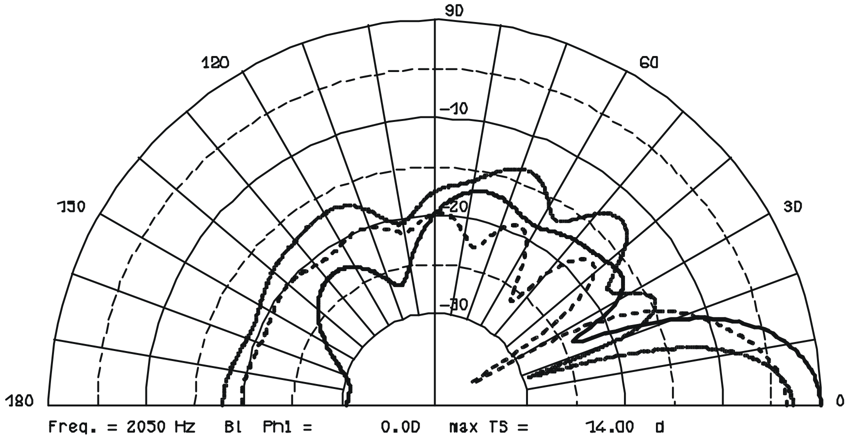


Figure V.9: Directivity patterns for example sph2d

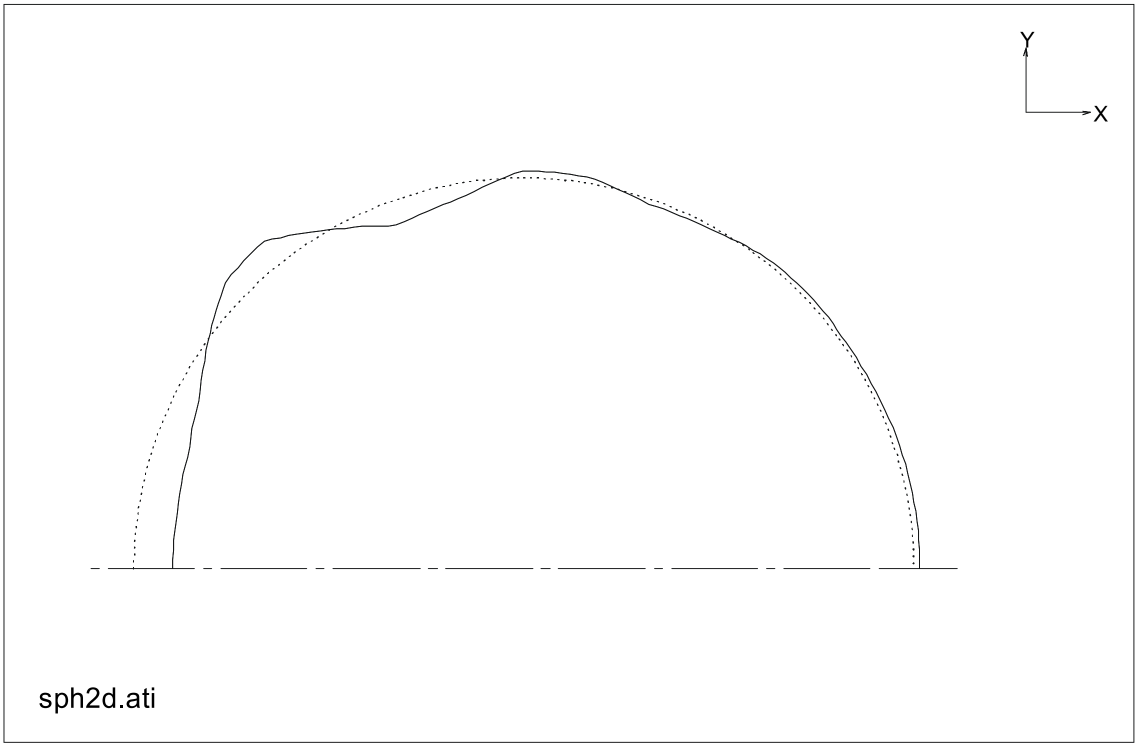


Figure V.10: Deformation of the shell in example sph2d

EQI

ISEN-LILLE

Appendices

Appendix I: Problem Equations

The problem to be solved is the propagation of acoustic waves in an infinite fluid medium containing one or more connex surfaces. The behavior of the structure formed by these surfaces is determined by the boundary conditions. The acoustic phenomena studied are limited to the dynamics of small perturbations of a compressible fluid.

The fluid considered is ideal, barotropic, irrotational, and with zero flow speed. The time dependence of the problem's descriptive variables is of the type $e^{-i\omega t}$ where ω is the system's frequency. Given the fluid properties, the knowledge of only one physical quantity is sufficient to model the problem. We select the excess pressure P. The fundamental wave propagation equation is:

$$\Delta P(\underline{r},t)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}P(\underline{r},t)=-\delta(\underline{r},\underline{r}_0,t,t_0)$$

where c is the free field propagation speed of the sound waves, P is a function of time t and spatial coordinates of point $\underline{r}$, δ is Kronecker's symbol, $\underline{r}_0$ is the position of the source, and t_0 the reference time. In the case of a radiation problem, the right-hand term is zero. In harmonic regime, the time and coordinate variables can be separated as follows:

$$P(\underline{r},t)=p(\underline{r})e^{-i\omega t}$$

The combination of these two equations gives Helmholtz' equation:

$$\Delta P(\underline{r},t)+k^2p=-\delta(\underline{r},\underline{r}_0)$$

where k is the wave number such that:

$$k=\frac{\omega}{c}$$

This equation must be completed with the boundary conditions at the surface of structure Γ. We can have:

- A Dirichlet condition:

$$p(\underline{r})=f(\underline{r})\quad \underline{r}\in\Gamma$$

- A Newmann condition:

$$\frac{\partial}{\partial n} p(\underline{r}) = g(\underline{r}) \quad \underline{r} \in \Gamma$$

where ∂p(r)/∂n is the normal derivative positively oriented outwards p at point $\underline{r}$ of Γ, and functions f and g are known. The **EQI** code takes into account only Newmann conditions through the normal displacement u_n, linked to the normal displacements by:

$$\frac{\partial}{\partial n} p(\underline{r}) = \rho\omega^2 u_n(\underline{r}) \quad \underline{r} \in \Gamma$$

A last condition that must be imposed is Sommerfield's conditions:

$$\lim_{r\to\infty}(r p_d(\underline{r})) \ \text{finite} \qquad \lim_{r\to\infty} r\left(\frac{\partial}{\partial r} p(\underline{r}) - ikp(\underline{r})\right) = 0$$

where r is the radius of the vector linking $\underline{r}$ to the coordinate system's origin. This condition implies that the energy flux passing through a sphere centered on the structure and with a positive radius is positive and finite.

Note: In the case of the problem of an incident wave scattering on the structure, it is Helmholtz' inhomogeneous equation, whose right-hand term contains the effect of the incident source, which is to be considered.

Appendix II: Auto-Scattering Coefficients and Radiation Impedance

The computation of the auto-scattering coefficient and radiation impedance is carried out in the post-processing phase.

Auto-Scattering Coefficients

The Auto-Scattering coefficient is:

$$D = \frac{1}{|p_i|} \iint_{\Gamma} \left(p_i(\underline{r}) + p_{dr}(\underline{r})\right)\overline{\beta}(\underline{r})d\underline{r}$$

where $|p_i|$ is the modulus of the reference incident pressure, p_i is the incident pressure of the scattering problem, p_{dr} is the rigid scattered pressure of the same scattering problem, and $\overline{\beta}$ is the complex conjugate of the displacement v($\underline{r}$) normalized by a reference speed V such that:

$$\beta(\underline{r}) = \frac{v(\underline{r})}{V}$$

The p_{dr} term is computed when the structure's surface Γ is assumed to be perfectly rigid (homogeneous Newmann condition). The displacement field β is assumed to be known.

Radiation Impedance

The radiation impedance is:

$$Z = \frac{1}{V} \iint_{\Gamma} p(\underline{r})\overline{\beta}(\underline{r})d\underline{r}$$

where p is the pressure solution of the radiation problem created by the displacement field whose conjugate normalized value is $\overline{\beta}$.

Appendix III: Point-Source Test

The point-source test is essential to validate the solution model when no other results (measurements, analytical, numerical) are available. The objective is to solve the analytical radiation associated with the same surface mesh and the same frequency but from one or several sources located inside the geometry, each radiating an analytical pressure of the form:

$$p_a(\underline{r}) = A\frac{e^{ikD}}{D}$$

where D is the distance from the source located at point r_0 to the observation point r, and A is any given amplitude. The value of the normal derivative of the pressure at the surface is then:

$$\frac{\partial}{\partial n} p_a(\underline{r}) = A\frac{e^{ikD}}{D^2}\left(ik - \frac{1}{D}\right)(\underline{r} - \underline{r_0}) \cdot \underline{n}$$

where $\underline{n}$ is the unit vector normal to the surface. By introducing these values in the **EQI** data file, we compute the surface pressures that solve the equation:

$$[A]\{p\}p_a = [B]\left\{\frac{\partial p_a}{\partial n}\right\}$$

These are then compared to the analytical pressures obtained from the first equation. To do so, the average quadratic error is defined:

$$EQM = \sqrt{\frac{\sum_{n=1}^{N} \left|p(\underline{r}_n) - p_a(\underline{r}_n)\right|^2}{\sum_{n=1}^{N} \left|p_a(\underline{r}_n)\right|^2}}$$

where $\underline{r}_n$ is on of the N mesh points. If we define a distribution of the point sources verifying the symmetries of the actual loading, a comparison to an analytical solution allows to validate the numerical treatment of the integral equations, and more precisely, to locate the presence of an irregular frequency.

Appendix IV: Frequency Interpolation

In the **EQI** code, the frequency interpolation only applies to the Helmholtz-Kirchhoff formulation. The principle consists of obtaining, by linear interpolation, the first and second terms of the linear system resulting from the discretization of the integral equations at a frequency theoretically comprised between two frequencies f_1 and f_2 for which the matrices have already been assembled.

To linearly interpolate, we must remove the oscillating nature of the functions that must be integrated in Helmholtz' integrals, i.e. in the e^{ikD} term that appears in Green's function associated to the infinite space, or its derivative, with:

$$D = \left|\underline{r} - \underline{r}'\right|$$

$\underline{r}'$ being a point along the integration surface, and $\underline{r}$ the computation point of the integral equation. During the assembly phase, and to eliminate the oscillating nature of the complex exponential function, we must minimize the quantity D. In the case of the three-dimensional model, we introduce a distance D_{ij} close to D and such that:

$$D_{ij} = \left|\underline{r} - \underline{r}_{ij}\right|$$

where r_{ij} is the i^{th} node in the topology of element Γ_j. From this, it can be found that:

$$e^{ikD} = e^{ikD_{ij}} e^{ik\left(D - D_{ij}\right)}$$

The term $e^{ikD_{ij}}$ being independent from the integration point, can be removed from Helmholtz' integral. The matrices thus obtained are written $[\hat{A}]$ and $[\hat{B}]$ and, having lost their oscillating nature, they can be linearly interpolated coefficient after coefficient between the two frequency bounds. In the case of the axially symmetric model, the process is more complex because it requires fictitious nodes placed at the circumference and separated by a distance equal to λ/4.

Note: It is sufficient to interpolate between the bounds corresponding to the null frequency (characterizing the behavior of an incompressible fluid obeying Laplace's equation) and the one corresponding to the maximum frequency of the mesh (dictated by the λ/4 criterion) to guarantee this method's efficiency. This choice of frequency bounds allows the automation of the interpolation technique.

Appendix V: Information for the User

This appendix is divided into two parts: the first gives, as an example, the classical values of some important parameters of the problem; the second contains the major quantitative limitations of the **EQI** code.

Parameter Values

If the computer platform allows it, the **ALGORITHM MEMORY** command is recommended.

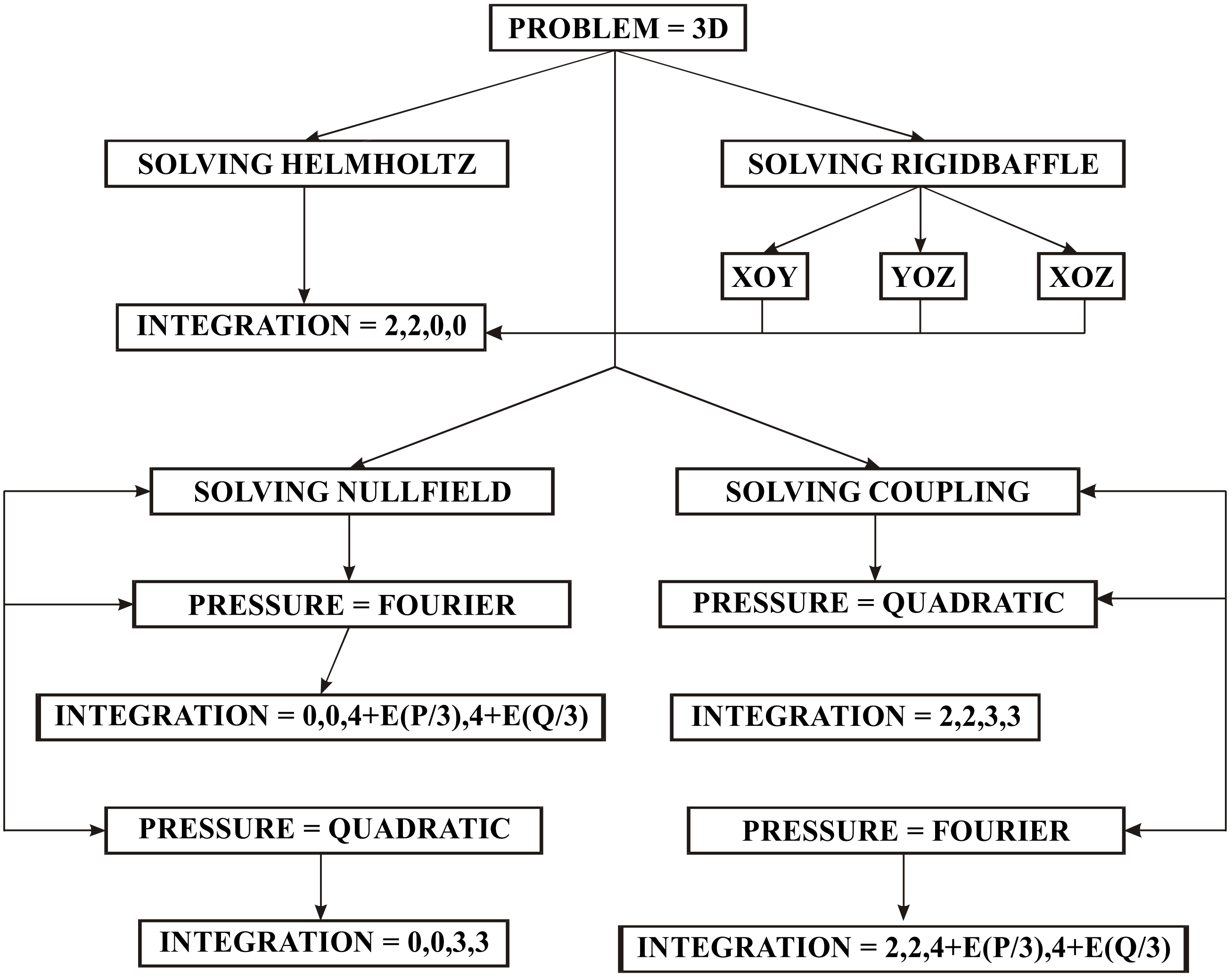
A printing level defined by **PRINTING = 0** is sufficient for a simulation "without problem."

The parameters of the **INTEGRATION** command vary according to the model and require that the $\lambda/4$ criterion be respected. According to the parameters of the **PROBLEM** command, three configurations are possible and are described below (the number of integration points of the null-field integrals is given for **MAXDEGREE < 10** and **E** designates the integer part).

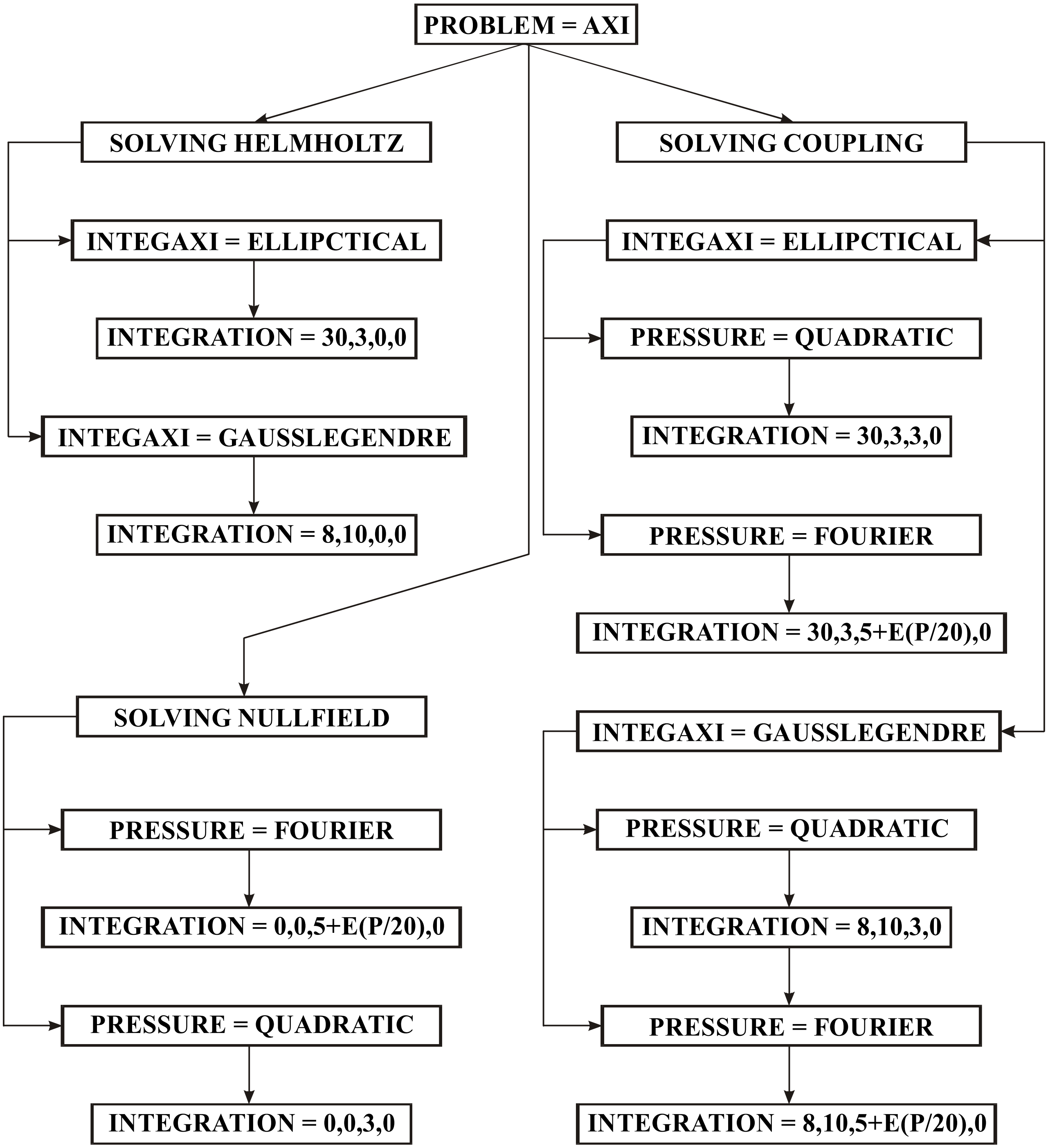
In the following illustrations, the parameters **MAXDEGREE**, **P**, **Q**, and **N** refer to those defined in Chapter III.

The **RESULTS=ANGULAR** command is recommended to obtain a better readability of the results file.

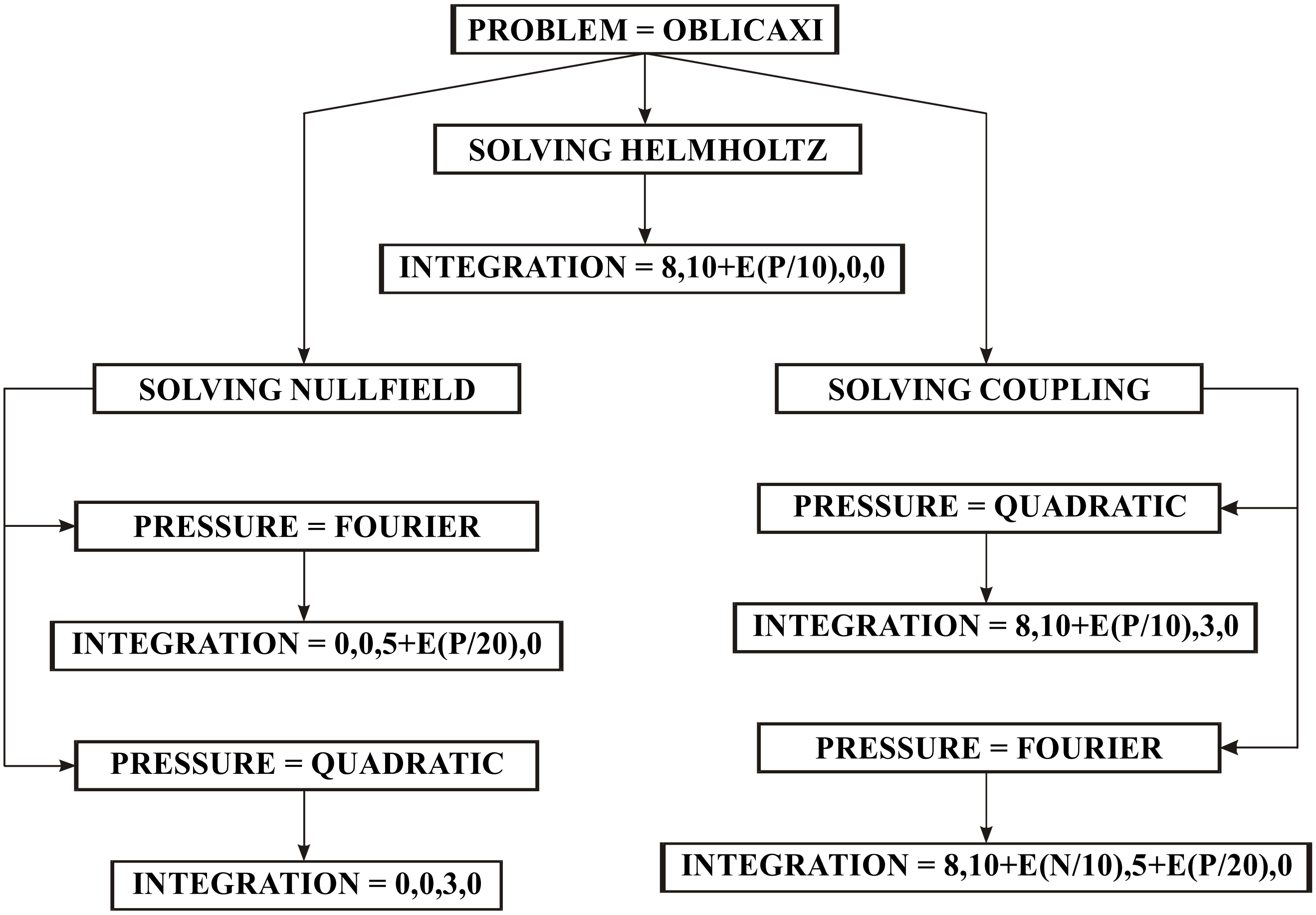
Configuration 1:



Configuration 2:



Configuration 3:



Quantification of the Limitations

The main dimensioning variables are contained in the **EQISIZ** file:

- The **ALGORITHM MEMOR**Y is available as long as the number of mesh nodes is less than **MAXMAT**. Otherwise, it is an algorithm swapping on the hard disk that must be used. In this last case, the maximum number of nodes is given by **MAXNDS**. The number of nodes that must be taken into account is that of the active part of the mesh when the decomposition in elementary problems is used.
- The number of loadings that can be treated simultaneously is limited to **MAXCHG**.
- The number of point sources that can be simultaneously taken into account is limited to **MAXSOU**.
- The number of incident waves that can be simultaneously taken into account is limited to **NODMAX**.
- The maximum number of polar diagrams at finite distance or in the far field is limited to **NTMDIR**.
- The maximum number of points in each polar diagram is limited to **MAXDIR**. In the case of a monostatic diagram, this number also defines the number of incidence directions.
- The maximum number of points that can be accepted in the case of the **POINTS** parameter of the **FLUID** command is limited to **NMPNT**.
- The maximum number of null-field equations is limited to **MAXCNU**.
- The maximum number of coefficients in the Fourier series decomposition is limited to **MAXCOF**.
- The maximum number of computation origins in the null-field equations is limited to **NORGMX**.
- The maximum number of integration points is limited to **MAXIN** when **PROBLEM = 3D** and **MAXPG** otherwise.